# Is the shape of the air shower front useful for researching the nature of cosmic rays? 

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## Preface

The study Applied Physics, which is taught at the Haagse Hogeschool in Delft, contains an graduate internship for 17 weeks. This internship is completed at the Institute for Mathematics, Astrophysics and Particle Physics (IMAPP) in Nijmegen. It would not possible if C. Timmermans did not give me this opportunity and for this I want to thank him. I also want to thank him for the accompaniment and help he gave. G. van Aar, S. Grebe, K. Holland, M. Lanfermann and H. Schoorlemmer helped me also during my internship with the LINUX system, which was new for me, and with the data analysis. I want to thank them too.

## VSummary

For many years scientists did not know where the ionizing radiation, measured in the atmosphere, came from. Between 1911 and 1913 Victor Francis Hess performed some measurements during balloon flights. When the balloon flew higher than 1.5 km above ground level, he measured an increase in the amount of ionizing radiation ${ }^{[1,2]}$. Robert Andrews Millikan assumed in 1925 that the radiation came from outer space and called this radiation "cosmic rays".

Cosmic rays are charged particles which permanently hit and penetrate the atmosphere of the Earth ${ }^{[1,2]}$. They will at a certain moment collide with nuclei in the atmosphere, like nitrogen. The results of such an interaction are new particles, which are called secondary particles. Every secondary particle will decay or interact again with the atmosphere. This process results in a cascade of particles, which is called an extensive air shower. At a given moment the shower becomes less dense, because the average particle energy is not enough to create more particles. The depth where the number of charged particles is on its maximum is called the shower max.
Every primary particle interacts differently with the atmosphere and thus causes a different air shower. For example, the density of the atmosphere has an influence on the production rate of secondary particles. A light/small primary, such as a proton, on average interacts later in the atmosphere than a heavy/big primary, such as an iron nucleus, and therefore the shower max is higher in the atmosphere for heavy particles.

The Pierre Auger Cosmic Ray Observatory in western Argentina detects cosmic rays with surface detectors and fluorescence detectors ${ }^{[1,2]}$. The surface detector detects the particles which reach the ground, using large tank filled with water. The Cherenkov radiation, which is emitted by particles that move faster than speed of light in water, is detected by photomultipliers. The fluorescence detector detects the ultraviolet light of the molecules in the atmosphere that fall back to the ground state after being excited by a secondary particle.

The Pierre Auger Collaboration reconstructs the data measured by these detectors and analyzes them. There are two reconstruction packages: Offline and CDAS, which contains different reconstruction algorithms. The exact difference between them is not clear but the parameters which are reconstructed are compared for air showers for which the primary particle has an energy above 30 EeV and an angle, at which the shower penetrates the Earth (measured from the vertical), of at most $60^{\circ}$. There is a median difference in the energy of $(-2.84 \pm 0.12) \mathrm{EeV}$ between the packages. This is due to the fact that the output signal of the detectors is calculated differently. A different in charge calibration, a different correction for saturated photomultipliers and a different start time for the measurements are the main reasons for this difference. The radius of curvature of the shower front and the direction of the air shower, which is represented by a zenith and azimuth angle, are reconstructed by the packages too. The median differences are respectively $(0.00 \pm 0.04) \mathrm{km}$, ( $0.069 \pm 0.007$ ) deg and ( $-0.032 \pm 0.016$ ) deg.

For the research to the nature of the cosmic rays, the mass of the primary particle has to be known. This can be calculated from the fluorescence detectors using the shower max. But these detectors only have a duty cycle of about $10 \%{ }^{[2]}$. The surface detectors have a duty cycle of $100 \%$. To get enough data to perform composition research at high energies, a parameter reconstructed from the information obtained from the surface detectors which is mass sensitive is needed. This parameter is the radius of curvature of the shower front. Using good quality reconstructed showers and a correction for the angle at which the shower penetrates the Earth, the radius of curvature correlates with the depth of the shower maximum and is therefore mass sensitive. The correlation factors for the Offline package for energies of the primary between 3 and 6 EeV becomes -0.34 . For

6-12 EeV: -0.25. For 12-24 EeV: -0.15 and for $24-48 \mathrm{EeV}:-0.66$. For CDAS this factors become respectively: $-0.31,-0.32,-0.33$ and -0.56 . The number 1 represents a linear functional relation and 0 represents no correlation. To explain the differences, more research must be done with respect to the differences between the packages.

The reconstructed value of the depth of the shower maximum from the fluorescence detector is compared to the calculated parameter using the radius of curvature from the surface detector. For energies above 6 EeV for Offline and 4 EeV for CDAS the calculated parameter is in agreement with the reconstructed parameter. Below these energy values the calculated parameter is lower. A reason for this effect can be an overestimation of the correlation between the curvature of the shower front and the shower maximum, which results in lower points.
At approximately 25 EeV for Offline and 13 EeV for CDAS the calculated shower maximum does not rise with energy as fast as before this energy. It is possible that the average mass of the cosmic rays become higher at high energy, because in that case the depth of the shower maximum is smaller. It can be concluded that the type of reconstruction package has an influence on the physics.

The zenith corrected radius of curvature, $R_{0}$, also increases as a function of energy for CDAS and Offline. When a primary particle has a higher energy, it will interact earlier in the atmosphere and therefore $R_{0}$ becomes bigger. This is an indication that the radius of curvature has a correlation between the height of the first interaction, which can still be investigated using simulations.
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## 21. Introduction

For many years scientists did not know where the ionizing radiation, measured in the atmosphere, came from. Between 1911 and 1913 Victor Francis Hess performed some measurements during balloon flights. When he increased the distance to the ground up to 1.5 km , he measured a decrease in the amount of ionizing radiation. But when the balloon flew higher, he measured an increase. His conclusion was that there was radiation coming from above ${ }^{[1,2]}$. Robert Andrews Millikan showed in 1925 that the decrease is due to the shielding of the radiaton. He assumed that the radiation came from outer space and called this radiation "cosmic rays".

### 1.1 Cosmic rays

Cosmic rays are charged particles which permanently hit and penetrate the atmosphere of the Earth. The energy of cosmic rays covers a range from $10^{9}$ to beyond $10^{20} \mathrm{eV}$. Beneath about $10^{16} \mathrm{eV}$, the energies of the particles are likely due to the acceleration in magnetic fields of several sources, such as the sun, solar winds or remnants of supernovae. There is a limit to the amount of energy a source can provide, because the magnetic fields cannot capture the particles inside the acceleration region infinitely long when the energy gets to be too high.

The origin of the high energy cosmic rays, with energies between $10^{16} \mathrm{eV}$ and $10^{18} \mathrm{eV}$, is uncertain, but it is generally believed to be from within our Milky-Way. Cosmic rays with energies above $10^{18} \mathrm{eV}$ are called Ultra-High-Energy-Cosmic-Rays (UHECR), and are most likely extra galactic. Active galactic nuclei are the most likely candidates for the sources of the highest-energy cosmic rays that hit Earth. These nuclei are compact regions at the centre of galaxies that have a much higher than normal luminosity over at least some portion, and possibly all, of the electromagnetic spectrum. This radiation is believed to be a result of accretion of mass by the super massive black hole at the centre of the host galaxy ${ }^{[3]}$.
In a given period of time there are more low energy than high energy and ultra high energy cosmic rays colliding with the atmosphere, see figure 1.
The relation between the flux and the energy can be approximated by a power law ${ }^{[1]}$ :

$$
\begin{equation*}
\frac{\mathrm{d} I}{\mathrm{~d} E} \propto E^{-\gamma} \tag{1}
\end{equation*}
$$

Where:

| $I$ | Flux | $\left(\mathrm{m}^{-2} \mathrm{~s}^{-1} \mathrm{sr}^{-1}\right)$ |
| :--- | :--- | :--- |
| $E$ | Energy | $(\mathrm{GeV})$ |
| $\gamma$ | Spectral index | $(-)$ |

The value of $\gamma$ is 2.7 for energies between $10^{11} \mathrm{eV}$ and $10^{15} \mathrm{eV}$ and above $10^{18} \mathrm{eV}$. Between $10^{15}$ and $10^{18} \mathrm{eV}, \gamma$ is 3.1.

The regions where $\gamma$ becomes different, are referred to the "knee" and the "ankle", see


figure 1. There is a cut off in the energy spectrum at about $10^{20} \mathrm{eV}$. This can be explained by the interaction of the cosmic rays with the cosmic microwave background. For example, a proton interacts with a photon according to:

$$
\begin{aligned}
& \mathrm{p}+\gamma \rightarrow \mathrm{p}+\pi^{0} \\
& \mathrm{p}+\gamma \rightarrow \mathrm{n}+\pi^{+}
\end{aligned}
$$

The threshold energy for these processes to occur is about $10^{20} \mathrm{eV}$. Therefore, the energy of cosmic rays higher than $10^{20} \mathrm{eV}$ will be reduced. This was first realized in 1966 by Kenneth Greisen, Georgiy Zatsepin and Vadim Kuzmin and therefore this threshold is called the GZK-limit. The cosmic ray composition between 50 EeV and 300 EeV is expected to be composed of mainly protons and iron nuclei, because these particles have the lowest chance to interact with the cosmic microwave background ${ }^{[5]}$.

### 1.2 Air showers

The primary cosmic rays penetrate the Earth and will at a certain moment collide with nuclei in the atmosphere, like nitrogen. The particle air cross section indicates the probability a particle collides with a nucleus. Heavy nuclei have a bigger particle air cross section than a proton. This means a proton, as cosmic ray, first interacts in lower atmospheres than a heavy nucleus of the same energy. The result of the interaction is the creation of several other particles, which are called secondary particles. Their total energy equals the energy of the primary particle. Every secondary particle will decay or interact again with the atmosphere. This process results in a cascade of particles, which is called an extensive air shower. Every primary particle interacts different with the atmosphere and causes a different air shower. The exactly components also depends on the circumstances, like the density of the atmosphere. Mainly, the air shower consists of 3 types of components:

## 1. Hadronic components

Hadrons are mostly created by interactions of hadrons with other hadrons. Most cosmic rays are atomic nuclei, which consists of hadrons like protons and neutrons. When they interact with nuclei in the atmosphere, heavy hadrons can be created. These decay or interact quickly, producing pions, kaons and other hadrons. These hadronic components are mostly located in the shower core, which is a result of the law of conservation of momentum. Low energy charged pions decay into muonic particles. Pions with zero electrical charge decay with a high probability into two photons. High energy photons generate an electromagnetic sub shower. All these interactions causes that only a small fraction of hadronic components reach the surface of the Earth. ${ }^{[1,2]}$

## 2. Electromagnetic components

Most particles in the shower belong to the electromagnetic component. Slightly more than a third of the energy goes into the electromagnetic components ${ }^{[6]}$. The high energy photons, which are the result of decay of hadronic components, undergo pair production ${ }^{[6]}$ :

$$
\gamma+\text { nucleus } \rightarrow \mathrm{e}^{+}+\mathrm{e}^{-}+\text {nucleus }
$$

The resulting electrons undergo bremsstrahlung in the electric field of atomic nuclei, thereby creating a high energetic photon:

$$
\mathrm{e}^{ \pm}+\text {nucleus } \rightarrow \gamma+\mathrm{e}^{ \pm}+\text {nucleus }
$$

Rutherford scattering is possible as well. These effects lead to a small lateral distribution of the air
shower. Therefore, mainly, the electromagnetic components are located next to the shower axis. Other electromagnetic effects, like the Compton effect, the photoelectric effect and ionization of the nuclei can be neglected at these high energies ${ }^{[6]}$.

## 3. Muonic components

Weak interaction of the hadronic components mainly results in muonic components:

$$
\begin{aligned}
& \pi^{+} \rightarrow \mu^{+}+v_{\mu} \\
& \pi^{-} \rightarrow \mu^{-}+\bar{v}_{\mu} \\
& \mathrm{K}^{+} \rightarrow \mu^{+}+v_{\mu} \\
& \mathrm{K}^{-} \rightarrow \mu^{-}+\bar{v}_{\mu} \\
& \mathrm{K}^{+} \rightarrow \pi^{+}+\pi^{0} \\
& \mathrm{~K}^{-} \rightarrow \pi^{-}+\pi^{0}
\end{aligned}
$$

They are mostly created in the early stages of the air shower development. Once muonic components are created, they hardly interact with the nuclei in the atmosphere. Therefore, they will move in a straight line with approximately the speed of light. Figure 2 shows an overview of the shower. All the interactions in the air shower take place until there is not enough energy to create new particles. The shower decays


Figure 2: Components of the air shower ${ }^{[7]}$. when this happens.

### 1.3 Detection

The Pierre Auger Cosmic Ray Observatory in western Argentina detects cosmic rays by two different methods. One method is to detect the particles that reach the ground. The other method detects the fluorescence light of the air shower. Both methods will be explained. The acquisition of the data from the detectors only start after a trigger had been given. This trigger will be explained as well.

### 1.3.1 Surface detectors

To detect the particles which reach the ground, water tanks are used. These tanks are also called stations. When a charged particle passes through the water with approximately the speed of light, it will go faster than the speed of light in water. Therefore Cherenkov radiation is emitted. This radiation is detected by three photomultipliers which are mounted at the upper side of the station in a regular manner, see figure 3. The output signal of the photomultipliers are given in VEM, which stands for Vertical Equivalent Muon. A muon which vertically and centrally passes the water, causes a signal equal to 1 VEM.

There are 1600 water tanks, in a total area of $3000 \mathrm{~km}^{2}$ with a spacing of 1500 m on a triangular grid, see figure 4 . Each


Figure 3: The surface detector, a water tank and electronics ${ }^{[8]}$.
tank has a diameter of 3.6 m and is filled with 12 tons of ultra pure water ${ }^{[1]}$. A solar panel is mounted on the top of the tank. This panel provides the energy for the electronic part of the station. When more energy is created than used, this energy can be stored in the battery. Thus the detector can be used during nights as well. This battery is mounted at the south side of the station, to protect it from direct sunlight. Each station has an antenna, to connect the tank to the wireless network of Auger. All tanks send their data to the central data acquisition system (CDAS). Via a GPS, the positions of the stations are determined within an accuracy of 1 m , and the timing of incoming particles is measured up to 20 ns .


Figure 4: Pierre Auger Observatory near the city of Malargüe, Argentinia. The red points represent the water tanks.
The area is overlooked by fluorescence telescopes. The angle between two green lines situated next to each other shows the fields of view of one fluorescence telescope ${ }^{[9]}$.

### 1.3.2 Fluorescence telescopes

A secondarv particle can excite the molecules in the atmosphere. When a molecule is excited, it can


[^0] fall back to the ground state and emit fluorescence ultraviolet (UV) light. This light can be detected by the telescopes, which overlook the sky above the water tanks. There are 4 buildings, also called eyes, with 6 telescopes each, see figure 4 . Each telescope has a field of view of $30^{\circ}$ horizontal, and $1^{\circ}$ to $31^{\circ}$ vertically above the horizon. The fluorescence light is detected by an array of photomultipliers, which looks at a $12 \mathrm{~m}^{2}$ spherical mirror. Before the light reflects on the mirror it passes a diaphragm with an aperture of $3.8 \mathrm{~m}^{2}$ and an UV passing filter, which is transparent for wavelengths in a range of 300 nm to 400 nm . See figure 5 . The energy of a cascade must be estimated from the pulse heights of the output signal, but these signals cannot always be measured
well. The fluorescence light can only be seen during moonless nights. Also the air conditions have an influence on the signal. There are several systems which measures these conditions, like LIDARs (light detection and rangings) or atmospheric monitoring systems. The first one has to determine the aerosol content in the air backscatter, the second atmospheric conditions. Furthermore, the density and the temperature of the atmosphere are measured using balloon flights. Next to the fluorescence light, the telescopes also measure the Cherenkov radiation that is emitted because of particles in the air shower move faster than the speed of light in air. The measurements are corrected for this effect using an empirical relation. The other influences described above are corrected for this too, but high quality data taking is only possible in periods without rain and storm, as well as heavy clouds. This can be done around new moon. This results in a duty cycle of about $10 \%{ }^{[2]}$.

### 1.3.3 Trigger levels

For the surface detectors there are two local trigger levels (Trigger level 1 (T1) and 2 (T2)), the array trigger level (T3), the physics event trigger level (T4) and the quality trigger level (T5). The local trigger levels are provided by the electronics, installed in each surface detector. T1 indicates that a tank is hit by a particle. If there are more particles detected in a tank, it passes T2. The T3 trigger signals indicates that several tanks had a T2 at the same time ${ }^{[2]}$. T4 performs the real shower selection. T5 is stricter than T4: also events with edge effects are removed, as well as other unreasonable data. T 5 is separated into 5 sub levels, starting with sublevel 0 (see figure 6):


Figure 6: Seven surface detectors. The orange station contains the strongest signal. The black stations are neighboring stations, which forms a row. The green framed stations are the three stations with the highest signals.
0. Station e or f may be out of work, or more than one of all the stations may be out of work.

1. Station $a, b, c$ or $d$, may be out of work. Station $e$ and $f$ must be working.
2. The hexagonal row (black in figure 6) around the station which detect the strongest signal, must be working.
3. Two hexagonal rows around the station which detect the strongest signal, must be working.
4. Three hexagonal rows around the station which detect the strongest signal, must be working.

### 1.4 Scope of this thesis

When the mass of the primary particles of the air showers is known, it might be possible that the primary particles can be traced back to their source and more research to their nature can be done. The study of their hadronic interactions can be performed in more detail as well. If the source of cosmic rays is identified, it is important to know what kind of particles it produces in order to study its acceleration mechanisms. For all this, the mass of the primary has to be calculated from the data of the fluorescence or surface detectors from the Pierre Auger Collaboration. The depth at which the number of charged particles in the air shower is at its maximum, $X_{\text {max }}$, is a mass sensitive parameter, see figure 7. This figure shows the predictions for proton and iron for several hadronic interaction models ${ }^{[11]}$ and the measured values. The proton and iron lines are separated and therefore it can be concluded that $X_{\max }$ is mass sensitive.


Figure 7: The value of $X_{\max }$ as a function of the energy of the primary particle. Data points are shown with the predictions for proton and iron for several hadronic interaction models ${ }^{[11]}$.

This value is reconstructed from the fluorescence telescope data, but these telescopes only have a duty cycle of about $10 \%$ as described above. The surface detectors have a duty cycle of $100 \%$. To get enough data to study the composition, a parameter reconstructed from the information obtained from the surface detectors which is mass sensitive needs to be constructed. The aim is to make figure 7 again with this parameter and compare the differences. How this is done and the result is shown in chapter 3.
Chapter 2 introduces the reconstructed parameters and the data which are used to perform the research. Here, the values of the parameters and the differences between two reconstruction packages will be explained too.

## 2. Reconstruction

A measured air shower is called an event. When reconstructing the air shower, parameters that characterize the event, such as the direction, arrival time, energy, and so on, are obtained. An event can be reconstructed by two different reconstruction packages. The parameters and the packages will be explained in this chapter.

### 2.1 Parameters

This paragraph explains the azimuth angle, the zenith angle, the radius of curvature, the shower maximum, the energy and $S(1000)$. It will be explained how these parameters are obtained from the data of the fluorescence detectors and the surface detectors. Two additional timing parameters will be explained first.

### 2.1.1 Arrival time $\left(\boldsymbol{t}_{\boldsymbol{i}}, \boldsymbol{t}_{\mathbf{0}}\right)$

The parameter $t_{0}$ is called the start time and is defined as the arrival time of the shower core on the ground. The arrival times, $t_{i}$, of the shower front at each station, $i$, are used to calculate the arrival direction of the shower axis, and therefore the direction of the primary cosmic rays.

### 2.1.2 Direction of the shower $(\boldsymbol{\theta}, \varphi)$

The direction of the shower axis is represented by an azimuth angle ( $\varphi$ ) and a zenith angle ( $\theta$ ). The zenith angle is defined as the angle between the vertical and the shower axis. The azimuth angle is defined as the angle between East and the shower axis. See figure 8. If the shower axis originates from the North, the azimuth angle $\varphi$ is $+90^{\circ}$.

To obtain the direction of the air shower from the surface detector data, the shower core position, the arrival times of the shower at each station and a spherical model of the shower front, which will be explained in chapter 2.1.3, are used. The coordinates of the shower core, $x_{\text {core }}$ and $y_{\text {core }}$, are calculated from the center of mass of all triggered station positions, $x_{i}$ and $y_{i}$, weighted by the square root of their signal strengths.

The fluorescence detector and at least one surface detector can also be used to determine the direction of the shower. The light which originates from the beginning of the shower will be earlier detected than light which is emitted later in the shower development. Using the fluorescence data and one surface detector near the shower core, the shower core position can be obtained. From the arrival times of the light at the fluorescence detector and the shower core position the azimuth and zenith angle can be obtained.


Figure 8: Defining the zenith angle
$(\theta)$ and the azimuth angle, $(\varphi)$. The red line represents the shower axis.


Figure 9: Shower development: radius of curvature and the first interaction ${ }^{[2]}$.

### 2.1.3 Radius of curvature ( $\boldsymbol{R}_{\boldsymbol{c}}$ )

High energy muons move approximately in a straight line to Earth as described before. Because a straight line is the shortest path, muons which are created in the beginning of the shower development form the front of the shower ${ }^{[2]}$. This front is approximately hemispherical and expands towards the Earth, see figure 9 . The parameter $R_{c}$ is defined as the radius of curvature of this shower front at the moment that the core of the shower hits the Earth' surface. It should be noted that the distance of the first interaction to Earth, $X_{1}$ is not exactly the same as the radius of curvature, $R_{c}$.

With the surface detector data a curvature fit can be performed, see figure 10. From this fit the value of the radius of curvature, $R_{c}$, is determined. The plot shows the arrival time of the shower at each stations, which is at a distance $r_{\text {axis }}$ of the shower axis. The curvature fit is corrected for the zenith angle, $\theta$, the azimuth angle, $\varphi$, and the start time $t_{0}$. This means that the measurements are turned into values which corresponds to the values in the plain perpendicular to the shower axis.


Figure 10: An example of a curvature fit. The parameter $t$ is defined as the arrival time at the ground in relation to the arrival time of the shower core.

### 2.1.4 Maximum of the shower $\left(X_{\text {max }}\right)$

The parameter $X_{\max }$ can be measured directly with the fluorescence telescopes. The amount of fluorescence light is proportional to the path length of the charged particles. Therefore, the most light will be emitted when there are the most charged particles. When, due to the limited field of view of the detector, only a part of the longitudinal shower profile is observed, an appropriate function is needed for the extrapolation to unobserved depths. This function is defined by Thomas K.

Gaisser and Anthony M. Hillas and is called the Gaisser-Hillas function ${ }^{[12]}$ :

$$
\begin{equation*}
f_{G H}(X)=N(X)=N_{\max }\left(\frac{X-X_{0}}{X_{\max }-X_{0}}\right)^{\frac{X_{\max }-X_{0}}{\lambda}} e^{\frac{X_{\max }-X_{0}}{\lambda}} \tag{2}
\end{equation*}
$$

Where:

| $N$ | Number of particles in the shower at depth $X$ | $(-)$ |
| :--- | :--- | :--- |
| $N_{\max }$ | Maximum number of particles in the shower | $(-)$ |
| $X$ | Depth | $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |
| $X_{0}$ | Energy dependent shower parameter, which does not have | $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |
| $X_{\max }$ | physical meaning ${ }^{[2,12]}$ | Depth at which the number of charged particles is at its maximum |
| $\lambda$ | Mean interaction length | $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |
| $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |  |  |

Using the data and (2), $X_{\max }$ can be calculated, see figure 11 and 12.

Gaisser Hillas fit, including datapoints


Figure 11: Gaisser Hillas fit (red), including the datapoints (black). The red cross displays the $X_{\max }$ and its uncertainty.

Gaisser Hillas fit, including datapoints


Figure 12: Gaisser Hillas fit (red), including the datapoints (black). The red cross displays the $X_{\max }$ and its uncertainty.

The value of $X_{\max }$ is proportional to the logarithm of the energy of the primary particle ${ }^{[13]}$, see appendix I for the derivation of the follow equation:

$$
\begin{equation*}
X_{\max } \propto \ln \left(\frac{E_{n u c l}}{A}\right) \tag{3}
\end{equation*}
$$

Where:
$E_{\text {nucl }}$
A
Energy of the nucleus
(eV)
(-)

This means that a heavy nucleus, like iron, has a smaller value of $X_{\max }$ when it has the same energy as a primary proton. See figure 7 again.

Also the muon fraction of the shower shows if the primary particle is heavy or not. A higher muon fraction is the result of a heavier primary particle ${ }^{[2]}$.

### 2.1.5 Energy $(E)$ and $\boldsymbol{S}(\mathbf{1 0 0 0})$

To determine the energy $(E)$ of the cosmic ray from the surface detector data, three steps have to be taken. First the signal at a distance of 1000 m from the core position, $S(1000)$, has to be estimated from the data of the surface detector. At a distance of 1000 m from the shower axis, the signal is almost independent of the primary mass and shower to shower fluctuations ${ }^{[14]}$. These fluctuations are due to the fact that the same primary particle with the same energy can result in a different shower, due to the stochastic nature of the collisions. However, $S(1000)$ depends on zenith angle $(\theta)$. When this angle is large, the distance through the atmosphere to the ground is larger. There will be fewer particles on the ground and therefore less signal. $S(1000)$ can be corrected to an equivalent density for showers arriving at the most frequent angle: $38^{\circ}$. This results in $S_{38^{\circ}}$. After that, the energy $(E)$ can be calculated using the correlation between $E$ and $S_{38^{\circ}}$, found in hybrid events. These are the events measured simultaneously with the surface detector and the fluorescence detector.

The first step can be completed using a lateral distribution fit. Figure 13 shows an example. This is a fit where the signal of the station, $S$, is plotted as a function of the perpendicular distance of the station to the shower axis, $r_{\text {axis }}$. High-developing showers have a flat lateral distribution, whereas low-developing showers produce steep lateral distributions.

## Lateral distribution fit



Figure 13: The signal of the surface detector, $S$, with a unit VEM, against the perpendicular distance to the shower axis, $r_{\text {axis }}$. The dark blue squares are the stations which detect the signal $S$. The light blue triangle displays a detector which was not used in the fit.

The lateral distribution fit is also corrected for the zenith angle, $\theta$, the azimuth angle, $\varphi$, and the start time $t_{0}$.

The signal as a function of the distance to the shower axis is described as follows ${ }^{[15]}$ :

$$
\begin{equation*}
S\left(r_{\text {axis }}\right)=S(1000) \cdot f_{L D F}\left(r_{\text {axis }}\right) \tag{4}
\end{equation*}
$$

Where:
$S\left(r_{\text {axis }}\right) \quad$ The signal at distance $r_{\text {axis }}$ from the shower axis (VEM)
$S(1000) \quad$ The signal at a distance of 1000 m from the shower axis
$f_{L D F}\left(r_{\text {axis }}\right)$ The parameterization of the shower shape where $f_{L D F}(1000)=1 \quad(-)$
$r_{\text {axis }} \quad$ Distance from the shower axis
For Auger, the function $f_{L D F}\left(r_{\text {axis }}\right)$ is described as ${ }^{[15]}$ :

$$
\begin{equation*}
f_{L D F}\left(r_{a x i s}\right)=\left(\frac{r_{a x i s}}{1000}\right)^{\beta}\left(\frac{r_{a x i s}+700}{1700}\right)^{\gamma_{1}+\beta} \tag{5}
\end{equation*}
$$

Where:
$\beta$ One out of two parameters which optimizes the shape of the lateral distribution fit
$\gamma_{1}$ One out of two parameters which optimizes the shape of the lateral distribution fit
When the value of $S(1000)$ is known, $S_{38^{\circ}}$ can be determined according to ${ }^{[16]}$ :

$$
\begin{equation*}
S_{38^{\circ}}=\frac{S(1000)}{1+0.92 x-1.113 x^{2}} \tag{6}
\end{equation*}
$$

With:

$$
\begin{equation*}
x=\cos ^{2}(\theta)-\cos ^{2}\left(38^{\circ}\right) \tag{7}
\end{equation*}
$$

Where:
$\begin{array}{lll}S_{38^{\circ}} & \text { Zenith independent parameter of } S(1000) & \text { (VEM) } \\ \theta & \text { Zenith angle } & \left({ }^{\circ}\right)\end{array}$
The energy can be determined as follows ${ }^{[17]}$ :

$$
\begin{equation*}
E=(1.68 \pm 0.05) \cdot 10^{17} \cdot S_{38^{\circ}}{ }^{(1.035 \pm 0.009)} \tag{8}
\end{equation*}
$$

The parameters used in equation (6) till (8) are used in the CDAS reconstruction. This will be explained in the next paragraph.

### 2.2 Reconstruction packages and their differences

There are two reconstruction packages: CDAS (= Central Data Acquisition) Online and Auger Offline. For the surface detector, both packages can be used to reconstruct the event. The package Auger Offline can also calculate parameters out of the data of the fluorescence detector. When both packages are compared, some differences can be observed.

First, the VEM output signal of the photomultipliers from the surface detector reconstructed by CDAS does not correspond to the VEM signal from Offline. This originates mainly from the VEM charge calibration. The signal processed by the surface detector local station electronics is the final readout of a long chain depending on different parameters, such as the water quality, the gain of the PMT, the electronic gain of the dynode and anode amplifiers ${ }^{[18]}$. The key parameter is the average charge, deposited by a centered and vertical high energy muon which goes through a tank. The average
charge is measured indirectly by plotting the distribution of charge deposited by muons crossing in all directions ${ }^{[18]}$, see the 3 -fold graph in figure 14.

Charge histogram of signals seen by a surface detector


Figure 14: Charge histogram of a surface detector under the flux of atmospheric muons. ${ }^{[18]}$
The first hump in the 3-fold graph is due to the triggering. The second hump corresponds to the signal of single muons going through the tank. The VEM-graph contains events triggered by a muon telescope as vertical and centered.
It can be concluded that the peak position of the second hump in the 3-fold graph is located close to the position of the peak in the charge distribution of vertical muons. The method to calculate this last peak position is different for CDAS and Offline ${ }^{[19]}$.

There also is a difference in correcting for saturated events. Some events have such a high number of particles in the tank that the photomultipliers become saturated. The two reconstruction packages do not correct for this in the same way. Offline takes the non linearity of the photomultipliers into account, but CDAS does not ${ }^{[19]}$. This results also in a different VEM output signal. Other differences come from differences in the baseline calculations and the start time in the two packages ${ }^{[19]}$.

Next, the output parameters can be compared. The events from the CDAS file are used until the year 2011. The events should have an energy higher than 30 EeV and a zenith angle lower than $60^{\circ}$. Above this zenith angle, a different reconstruction algorithm is used, which shows some biases. The events which meets these requirements are compared with the same event reconstructed by Offline. Figure 15a shows the difference between the energy reconstructed by the two packages and figure 15 b the difference between the relative uncertainty of the energy.


Figure 15a: Energy differences between Offline and CDAS for events with $E_{C D A S}>30 \mathrm{EeV}$ and $\theta<60^{\circ}$.


Figure 15b: Differences in the relative uncertainty of the energy between Offline and CDAS, with $E_{C D A S}>30 \mathrm{EeV}$ and $\theta<60^{\circ}$.

To determine the most frequent difference in the energy, a Gauss curve is fitted around the maximum of the graph. It shows a maximum at $(-2.84 \pm 0.12) \mathrm{EeV}$. This means that the calculated energy in the package CDAS is on average larger than for the same event in Offline. For the relative uncertainty of the energy, the package Offline generally has a $-0.0147 \pm 0.0003$ bigger value than CDAS reconstructs. It should be noted that this does not mean CDAS is a worse package. The differences in energy may be the result of a different VEM signal. To conclude if this is true, $S(1000)$ has to be different too and the equations to get the energy out of $S(1000)$ has to be the same. Figure 16 a and b shows the graphs for $S(1000)$.


Figure 16a: $S(1000)$ differences between Offline and CDAS with
$E_{C D A S}>30 \mathrm{EeV}$ and $\theta<60^{\circ}$.
Figure 16b: Differences in the relative uncertainty of $S(1000)$ between Offline and CDAS with $E_{C D A S}>30 \mathrm{EeV}$ and $\theta<60^{\circ}$.

Also $S(1000)$ and the relative uncertainty of $S(1000)$ for Offline is smaller than for CDAS. The maxima are respectively $(-10.4 \pm 0.5)$ VEM and $-0.0058 \pm 0.0003$.

Figure 17 shows that the packages calculate the value of $E$ out of $S(1000)$ in the same way. Here, the same events for Offline and CDAS are compared. The fitted line of Offline equals the line of CDAS within the uncertainties. The differences in the number of entries is due to 7 entries just outside the histogram.


Figure 17: Calculating the energy $E$ from the value of $S(1000)$ for CDAS (blue line) and Offline (red line). The parameters $p_{0}$ and $p_{1}$ are the linear fit parameters: $S(1000)=p_{1}(E-60)+p_{0}$.

It can be concluded that the differences in energy is the result of a differences in the VEM signal.
Using equation (6) until (8), the energy over $S(1000)$ can be estimated as:

$$
\frac{E}{S(1000)} \approx \frac{1+0.92 \cos ^{2}(\theta)-0.92 \cos ^{2}\left(38^{\circ}\right)-1.113\left(\cos ^{2}(\theta)-\cos ^{2}\left(38^{\circ}\right)\right)^{2}}{0.168}
$$

Figure 17 takes all zenith angles into account. Figure 18 a to c shows a difference in the zenith angle correction for both packages, when the same events are compared.

Profile of energy/S(1000)


Figure 18a: Profile of the energy over $S(1000)$ as a function of the zenith angle.

CDAS: Energy/S(1000)


Figure 18b: 2D Histogram of the energy over $S(1000)$ for CDAS.

Offline: Energy/S(1000)


Figure 18c: 2D Histogram of the energy over $S(1000)$ for Offline.

It can be concluded that the function to calculate $S_{38^{\circ}}$ out of $S(1000)$, equation (6), and/or the function to calculate $E$ out of $S_{38^{\circ}}$, equation (8), is different for both packages.

Figure 19 a and b shows the differences between the two packages for the obtained radius of curvature, $R_{c}$.


Figure 19a: Radius of curvature differences Offline and CDAS.


Figure 19b: Differences between the relative uncertainty of the radius of curvature of Offline and CDAS.

The Gauss fits shows a maximum of respectively $(0.00 \pm 0.04) \mathrm{km}$ and $-0.0052 \pm 0.0005$. Therefore, the radius of curvature reconstructed by both packages are mostly the same.

To determine if CDAS defines a too large uncertainty in the $S(1000)$, and therefore also in the $E$, the $\chi^{2}$ test is required on the lateral distribution fit, see figure 13 . This can also be done with the curvature fit.

The value of $\chi^{2}$ can be determined as follows:

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{M} \frac{\left(X_{i}-\bar{X}\right)^{2}}{\sigma^{2}} \tag{9}
\end{equation*}
$$

Where:
$\chi^{2}$ Test variable (-)
$M \quad$ Number of detectors which detect a signal
$X_{i} \quad$ Measured value of the signal
$\bar{X} \quad$ Expected value of the signal
$\sigma \quad$ Uncertainty on the detected signal
Therefore $\left(X_{i}-\bar{X}\right)$ represents the deviation of the measured signal and the expected signal. Dividing $\chi^{2}$ by de number of degrees of freedom results in a value which represents the quality of the fit. The value 1 represents a good quality fit. Above this, the fit become worse.
The number of degrees of freedom can be read from the file of Offline, but cannot be read from the file of CDAS. Therefore another value has to replace this. The number of degrees of freedom equals the number of stations minus three, because there are three parameters which have to be calculated to define the expected fit. These three are the start time $t_{0}$, the azimuth angle $\varphi$ and the zenith angle $\theta$. Now, the histogram of the quality of the lateral distribution fits can be plotted. The events which are used have the following requirements:
$\left|R_{c, \text { Offline }}-R_{c, C D A S}\right|<5 \mathrm{~km} ;$
$\left|E_{\text {Offline }}-E_{C D A S}\right|<20 \mathrm{EeV}$;
The T5 trigger value is already in the Offline file. These cuts remove the out layers of the reconstruction. These out layers are shown in appendix II. The histogram of the quality of the lateral distribution fits for the events which meets the cuts can be plotted, see figure 20.


Figure 20 shows that for CDAS the $\chi^{2}$ - distribution peaks at a value larger than 1 . Such a situation can be due to an underestimate of the uncertainties of the parameter used in the lateral distribution fit and therefore in the energy. The graph in figure 15b will move further away from zero if the uncertainties used in CDAS increase.

Figure 21 shows the quality of the curvature fits. Here, the Offline curve peaks above 1. Therefore, the uncertainty of the radius of curvature is most likely small when using the Offline reconstruction. This will move the uncertainty comparison in figure 19b to the right.

Finally, for $\theta$ and $\varphi$, figure 22 and 23 are plotted. These are plots without the cuts described above figure 20.


Figure 22a: Zenith angle differences Offline and CDAS.

Figure 22b: Differences between the uncertainty of the zenith angle of Offline and CDAS.

Figure 22 a shows a difference in zenith angle to peak at $(0.069 \pm 0.007)$ deg. The median difference of the uncertainty of this parameter is $(-0.004 \pm 0.010)$ deg, see figure 22 b .


The median difference in the azimuth angle is $(-0.032 \pm 0.016)$ deg, see figure 23 a . For the differences in the uncertainty of this parameter a value of $(-0.021 \pm 0.006)$ deg is most frequently found, see figure 23 b. The peak in figure 23 b is not symmetric. This can be the result of bigger values of $\varphi_{C D A S}$ and also a bigger values of the absolute uncertainty.

If only the events are used with the cuts shown above figure 20, the median difference of the several parameters changes, see table 1. From the 897 events, 570 pass the cuts.

Table 1: Median difference of several parameters with(out) the cuts shown above figure 20.

| Parameter | Median difference without cuts | Median difference with cuts |
| :---: | :---: | :---: |
| $E_{\text {Offline }}-E_{C D A S}$ | (-2.84 $\pm 0.12) \mathrm{EeV}$ | (-2.6 $\pm 0.3) \mathrm{EeV}$ |
| $\frac{\Delta E_{\text {Offline }}}{E_{\text {Offline }}}-\frac{\Delta E_{C D A S}}{E_{C D A S}}$ | $-0.0147 \pm 0.0003$ | $-0.0141 \pm 0.0005$ |
| $S(1000)_{\text {Offline }}-S(1000)_{\text {CDAS }}$ | (-10.4 $\pm 0.5)$ VEM | $(-9 \pm 2)$ VEM |
| $\frac{\Delta S(1000)_{\text {offline }}}{S(1000)_{\text {Offline }}}-\frac{\Delta S(1000)_{\text {CDAS }}}{S(1000)_{\text {CDAS }}}$ | $-0.0058 \pm 0.0003$ | $(-0.00852 \pm 0.00114)$ |
| $R_{C, O f f l i n e}-R_{c, C D A S}$ | $(0.00 \pm 0.04) \mathrm{km}$ | $(-0.04 \pm 0.04) \mathrm{km}$ |
| $\frac{\Delta R_{c, \text { Offline }}}{R_{c, \text { Offline }}}-\frac{\Delta R_{c, C D A S}}{R_{c, C D A S}}$ | $-0.0052 \pm 0.0005$ | $-0.0054 \pm 0.0004$ |
| $\theta_{\text {Offline }}-\theta_{\text {CDAS }}$ | (0.069 $\pm 0.007$ ) deg | (0.0576 $\pm 0.0110$ ) deg |
| $\Delta \theta_{\text {Offline }}-\Delta \theta_{\text {CDAS }}$ | (-0.004 $\pm 0.010) \mathrm{deg}$ | (-0.0139 $\pm 0.0010) \mathrm{deg}$ |
| $\varphi_{\text {Offline }}-\varphi_{\text {CDAS }}$ | $(-0.032 \pm 0.016) \mathrm{deg}$ | $(-0.049 \pm 0.018) \mathrm{deg}$ |
| $\Delta \varphi_{\text {Offline }}-\Delta \varphi_{\text {CDAS }}$ | (-0.021 $\pm 0.006) \mathrm{deg}$ | (-0.023 $\pm 0.003) \mathrm{deg}$ |

According to this paragraph, the well reconstructed events fulfill the following requirements:

$$
\begin{aligned}
& \left|R_{C, \text { Offline }}-R_{C, C D A S}\right|<5 \mathrm{~km} ; \\
& \left|E_{\text {Offline }}-E_{C D A S}\right|<20 \mathrm{EeV} ; \\
& \left|\theta_{\text {Offline }}-\theta_{C D A S}\right|<1^{\circ} ; \\
& \left|\varphi_{\text {Offline }}-\varphi_{C D A S}\right|<2^{\circ}
\end{aligned}
$$

## /3. Radius of curvature as mass sensitive parameter

To get enough data for composition research a mass sensitive parameter from the surface detector is needed. Guus van Aar shows in his master thesis ${ }^{[2]}$ that the radius of curvature, $R_{c}$, of the shower front is a mass sensitive parameter. This chapter does the same, but with additional quality cuts to the reconstruction of the air shower. This chapter also includes a comparison of the reconstructed parameter $X_{\max }$ from the fluorescence telescope and the calculated parameter of $X_{\max }$, using $R_{c}$, from the surface detector data. All this is done both by the Offline reconstructed data and the CDAS reconstructed data.

### 3.1 Offline

This paragraph uses the data reconstructed obtained from 2004 until March 2011 and reconstructed with the Offline package.

### 3.1.1 Zenith correction

A shower that starts after a certain slant depth will have a larger radius of curvature when its zenith angle, $\theta$, is larger. The perpendicular distance between the (blue) point $R_{\text {start }}$ and Earth will increase with $\cos (\theta)$, see figure 24 .


Figure 24: Zenith correction for the radius of curvature. As $\theta$ increases, $R_{c}$ increases with $\cos (\theta)$.


Figure 25: Zenith correction for the radius of curvature. As $\theta$ increases, $R_{c}$ increases stronger than only according to figure 24.

The point where the primary particle first interacts will be further away from Earth when the zenith angle, $\theta$, is larger, see figure 25. Therefore, the dependence of $R_{c}$ on $\cos (\theta)$ becomes stronger. A zenith correction for this radius of curvature is needed to continue the research. If this corrected parameter is proportional to the value of $X_{\max }$, it can be concluded that $R_{c}$ is a mass sensitive parameter.

The relation between $\theta$ and $R_{c}$ can be written as ${ }^{[2]}$ :

$$
\begin{equation*}
R_{c}=\frac{R_{0}}{\cos ^{\alpha}(\theta)} \tag{10}
\end{equation*}
$$

Where:

$$
\begin{array}{lll}
R_{0} & \text { Corrected radius of curvature } & (\mathrm{m}) \\
\alpha & \text { Function of energy, which has to be determined. } \tag{-}
\end{array}
$$

To derive the function $\alpha(E)$ a $\left(R_{c}, \theta\right)$ graph is plotted for different energy bins. These bins are logarithmic in energy, ranging between 3 and 100 EeV . To obtain only the well reconstructed events
for this plot, some requirements are imposed on the Offline reconstructed data. Table 2 shows these requirements, also called cuts, on the data measured by the surface detector. The standard cuts are made in the master thesis ${ }^{[2]}$ too. The additional cuts are the result of chapter 2.2 and remove the events which have a reconstructed parameter which differs to much between Offline and CDAS.

Table 2: Cuts on the Offline reconstructed parameters measured by the surface detector.

| Standard cut ${ }^{[2]}$ | Motivation |
| :---: | :---: |
| Reconstruction level $\geq 4$ | Sometimes the reconstruction does not go well. For example, there are only 3 points in de lateral distribution fit. A level above 3 represents the good reconstructed events. |
| T5 sublevel 1 | This trigger level requires 5 tanks around the station, which detect the highest signal, must be working. |
| $E>3 \mathrm{EeV}$ | The surface detectors become fully efficient above 3 EeV . |
| $\theta<60^{\circ}$ | Above this zenith angle, a different reconstruction algorithm is used, which shows some biases. |
| $\frac{\Delta R_{c}}{R_{c}}>0.015$ | Some outliers are removed. |
| $\frac{\chi_{\text {LDF }}^{2}}{N_{d o f, L D F}}<3$ | To get only the good quality lateral distribution fit events. |
| $\frac{\chi_{\text {curv }}^{2}}{N_{\text {dof, curv }}}<8$ | To get only the good quality curvature fits events. |
| $N_{\text {dof, curv }}>1$ | Events with fewer than two degrees of freedom in the curvature fit show some artifacts in the distribution of $R_{c}$ and $d R_{c}$ |
| Bad Period ID = 0 | In some periods of time there are bad measurements. The events in this period have a bad period ID $>0$. |
| Additional cut | Motivation |
| $\left\|R_{c, \text { Offline }}-R_{c, \text { CDAS }}\right\|<5 \mathrm{~km}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $\left\|E_{\text {Offline }}-E_{C D A S}\right\|<20 \mathrm{EeV}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $\left\|\theta_{\text {Offline }}-\theta_{\text {CDAS }}\right\|<1^{\circ}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $\left\|\varphi_{\text {Offline }}-\varphi_{\text {CDAS }}\right\|<2^{\circ}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |

Events reconstructed by one package will not always be reconstructed by the second package, because they have different trigger levels. Due to the additional cuts, data is required to be reconstructed by both packages. This will remove some events too.

To plot the $\left(R_{c}, \theta\right)$ graph, the uncertainties of the radius of curvature and the zenith angle have to be described as well. For the uncertainty on the zenith angle, the reconstruction uncertainty is used. For $R_{c}$ this is not good enough. There are shower to shower fluctuations. This has to be included in the uncertainty when deducing a relationship for $R_{c}$ on $\theta$. How this is done is copied from the master thesis ${ }^{[2]}$. Because $R_{c}$ depends on the zenith angle and the energy, the shower to shower fluctuations also depends on these parameters. The distribution of the reconstructed values of $R_{c}$ in one $(\log (E), \cos (\theta))$ bin has an observed width, $\sigma_{o b s}$. This $\sigma_{o b s}$ is therefore the result of a combination of the distribution due to shower to shower fluctuations, $\sigma_{s-s}$, and the uncertainty on the reconstructed values of $R_{c}$. The standard deviation of the values of $R_{c}$ in a bin is taken as the
uncertainty on the value of an event. The value of $\sigma_{s-s}$ becomes:

$$
\begin{equation*}
\sigma_{s-s}^{2}=\sigma_{o b s}^{2}-\left\langle d R_{c}\right\rangle^{2} \tag{11}
\end{equation*}
$$

Where:

| $\sigma_{s-s}$ | Distribution width due to shower to shower fluctuations | $(\mathrm{m})$ |
| :--- | :--- | :--- |
| $\sigma_{o b s}$ | The RMS of the values of $R_{c}$ in an energy and zenith angle bin | $(\mathrm{m})$ |
| $\left\langle d R_{c}\right\rangle^{2}$ | Mean value of the uncertainties of $R_{c}$ in an energy and zenith angle bin | $\left(\mathrm{m}^{2}\right)$ |

For each event, the contribution of shower to shower fluctuations to the uncertainty of $R_{c}$ at the corresponding energy and zenith is added quadratically to the reconstructed uncertainty ${ }^{[2]}$ :

$$
\begin{equation*}
d R_{t o t}=\sqrt{d R_{r e c}^{2}+\sigma_{s-s}^{2}} \tag{12}
\end{equation*}
$$

Where:
$d R_{t o t}$
Total applied uncertainty on the value of $R_{C}$
(m)
$d R_{\text {rec }} \quad$ The reconstructed uncertainty on $R_{c}$
(m)

The resulting plots of $\left(R_{c}, \theta\right)$ are shown in figure 26a to j .


Figure 26a: Radius of curvature as a function of the zenith angle for an energy between 3 and 4.25 EeV. 7320 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$



Figure 26b: Radius of curvature as a function of the zenith angle for an energy between 4.25 and 6.05 EeV . 6578 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos p_{1}(\theta)}
$$



Figure 26c: Radius of curvature as a function of the zenith angle for an energy between 6.05 and 8.59 EeV .4618 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos p_{1}(\theta)}
$$



Figure 26e: Radius of curvature as a function of the zenith angle for an energy between 12.20 and 17.32 EeV . 1685 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1(\theta)}}}
$$



Figure 26 g : Radius of curvature as a function of the zenith angle for an energy between 24.60 and 34.92 EeV . 403 Events. The red curve is described by:
$R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}$.


Figure 26d: Radius of curvature as a function of the zenith angle for an energy between 8.59 and 12.20 EeV .3003 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1(\theta)}}}
$$



Figure 26 f : Radius of curvature as a function of the zenith angle for an energy between 17.32 and 24.60 EeV .888 Events. The red curve is described by:

$$
R_{C}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$



Figure 26 h : Radius of curvature as a function of the zenith angle
for an energy between 34.92 and 49.59 EeV .146
Events. The red curve is described by:
$R_{c}=\frac{p_{0}}{\cos ^{1}(\theta)}$.


Figure 26i: Radius of curvature as a function of the zenith angle for an energy between 49.59 and 70.42 EeV. 64 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$



Figure $\mathbf{2 6 j}$ : Radius of curvature as a function of the zenith angle for an energy between 70.42 and 100 EeV .16 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$

Each value of $p_{1}$ for the different energy bins equals $\alpha$ of equation (10), see figure 26. A plot of $\alpha$ as a function of $\log (E)$ results in the required function, see figure 27.


Figure 27: Fit results of the power $\alpha$ as a function of $E . \alpha=p_{2}(\log (E))^{2}+p_{1} \log (E)+p_{0}$
The function $\alpha(E)$ becomes:

$$
\begin{equation*}
\alpha(E)=(-0.10 \pm 0.07) \cdot(\log (E))^{2}+(0.05 \pm 0.15) \log (E)+(1.34 \pm 0.08) \tag{13}
\end{equation*}
$$

Without the additional cuts, the function $\alpha(E)$ was obtained as ${ }^{[2]}$ :

$$
\begin{equation*}
\alpha(E)=(-0.13 \pm 0.06) \cdot(\log (E))^{2}+(0.11 \pm 0.11) \log (E)+(1.32 \pm 0.06) \tag{14}
\end{equation*}
$$

It can be concluded that equation (13) equals equation (14) within its uncertainties.

Figure 28 shows that $\chi^{2} / N_{\text {dof }}$ of two of the fits in figure 26 are above 1 . Most likely, the uncertainties are underestimated.


For all the events reconstructed by Offline, $R_{0}$ is calculated using equation (10) and (13). The results are shown in figure 29. This figure shows that at higher energies the corrected radius of curvature increases. When a primary particle has an higher energy, it will interacts earlier in the atmosphere, which results in a larger corrected radius of curvature.

### 3.1.2 Correlation between $X_{\text {max }}$ and $\boldsymbol{R}_{0}$

Next the calculated parameter of $R_{0}$ has to be correlated with $X_{\max }$ to investigate if the value of $R_{0}$ is mass sensitive. The parameter $X_{\max }$ is reconstructed using data from the fluorescence telescope. For one event, one or more fluorescence telescopes can provide data. It is therefore possible that there are two or more values for $X_{\max }$ reconstructed for the same event. Then the average value will be calculated taking the uncertainty into account. A weight is calculated as ${ }^{[2]}$ :

$$
\begin{equation*}
w_{t e l}=\frac{1}{\left(\Delta X_{\max }\right)^{2}} \tag{15}
\end{equation*}
$$

Where:

| $w_{\text {tel }}$ | Weight | $\left(\mathrm{cm}^{4} / \mathrm{g}^{2}\right)$ |
| :--- | :--- | :--- |
| $\Delta X_{\text {max }}$ | Uncertainty of $X_{\max }$ | $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |

The average value becomes ${ }^{[2]}$ :

$$
\begin{equation*}
X_{\max , \mathrm{avg}}=\frac{\sum_{i=1}^{N} X_{\max , i} w_{t e l, i}}{\sum_{i=1}^{N} w_{t e l}, i} \tag{16}
\end{equation*}
$$

Where:

| $X_{\text {max }, \text { avg }}$ | Average $X_{\max }$ | $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |
| :--- | :--- | :--- |
| $X_{\max , i}$ | Value of $X_{\max }$ for telescope $i$ | $\left(\mathrm{~g} / \mathrm{cm}^{2}\right)$ |
| $w_{\text {tel }, i}$ | Value of $w_{\text {tel }}$ for telescope $i$ | $\left(\mathrm{~cm}^{4} / \mathrm{g}^{2}\right)$ |
| $N$ | Number of telescopes which detect the shower | $(-)$ |

The uncertainty on this quantity becomes ${ }^{[2]}$ :

$$
\begin{equation*}
\Delta X_{\max , a v g}=\sqrt{\frac{1}{\sum_{i=1}^{N} w_{\text {tel }, i}}} \tag{17}
\end{equation*}
$$

Where:

$$
\Delta X_{\max , a v g} \quad \text { Uncertainty of the average } X_{\max } \quad\left(\mathrm{g} / \mathrm{cm}^{2}\right)
$$

Actually, to determine a correlation between $R_{0}$ and $X_{\max }$ the values of $R_{0}$ and $X_{\max , a v g}$ are used. From now, the average value is called $X_{\max }$ again, with its corresponding uncertainty, $\Delta X_{\max }$.

The fluorescence golden hybrid events are used. This is a set of events that can be reconstructed independently from the measurements of both detectors. The cuts on these events, used to get high quality reconstructed data, are shown in table 3.

Table 3: Cuts on the fluorescence detector data reconstructed by Offline ${ }^{[2]}$.

| Standard cut | Motivation |
| :---: | :---: |
| Reconstruction level $\geq 10$ | A level above 9 for fluorescence detector data equals a level above 3 for surface detector data. |
| $\frac{\chi_{G H}^{2}}{N_{d o f, G H}}<3$ | The Gaisser-Hillas profile must be a good fit to the observed data. |
| $X_{\max }= \pm 50 \mathrm{~g} / \mathrm{cm}^{2} \text { and in }$ telescope field of view | Ensures that the fitted location of $X_{\text {max }}$ is correct. |
| Cherenkov fraction < 50\% |  |
| $\Delta X_{\text {max }}<40 \mathrm{~g} / \mathrm{cm}^{2}$ |  |
| $\frac{\Delta E}{E}<0.2$ | The relative uncertainty in the energy reconstructed by the fluorescence detector must be smaller than 0.2. |
| $E>3 \mathrm{EeV}$ | Equal to the surface detector cut to avoid biases. |
| $\theta<60^{\circ}$ | Equal to the surface detector cut to avoid biases. |
| $\frac{\chi_{\text {line }}^{2}-\chi_{G H}^{2}}{N_{d o f, G H}}>1$ | There must be a sufficient difference in fit quality between a linear fit and a Gaisser-Hillas profile to ensure that the fitted location of $X_{\max }$ is reliable. |
| Cloud coverage < 20\% | About half of the events have no LIDAR measurements. These are not removed, but when there is LIDAR data the cloud coverage needs to be less than $20 \%$. |
| VAOD at $3 \mathrm{~km}<0.05$ | The Vertical Aerosol Optical Depth is an indication for the attenuation of the fluorescence light. VAOD at 3 km is the vertical integration of the extinction coefficient from ground level to 3 km up. |
| $\frac{1}{N_{d o f}} \sum_{i=S D, F D \prime s} \frac{\left(E_{i}-\bar{E}\right)^{2}}{\sigma_{E_{i}}^{2}}<5$ | The events that pass all previous cuts have an independent energy estimation from the surface detector and the fluorescence detector. These measurement must be in agreement. |

The correlation between the two quantities $\left(R_{c}\right.$ and $X_{\max }$ ) can be represented by a correlation coefficient $r{ }^{[20,2]}$ :

$$
\begin{equation*}
r=\frac{\sum_{i=1}^{n} w_{i}\left(X_{\max , i}-\overline{X_{\max }}\right)\left(R_{0, i}-\overline{R_{0}}\right)}{\sqrt{\left(\sum_{i=1}^{n} w_{i}\left(X_{\max , i}-\overline{X_{\max }}\right)^{2}\right) \cdot w_{i}\left(\sum_{i=1}^{n}\left(R_{0, i}-\overline{R_{0}}\right)^{2}\right)}} \tag{18}
\end{equation*}
$$

With weights:

$$
\begin{equation*}
w_{i}=\frac{1}{\left(\frac{\Delta X_{\max , i}}{X_{\max , i}}\right)^{2}}+\frac{1}{\left(\frac{\Delta R_{0, i}}{R_{0, i}}\right)^{2}} \tag{19}
\end{equation*}
$$

Where:

| $r$ | Correlation coefficient | $(-)$ |
| :--- | :--- | :--- |
| $n$ | Number of events | $(-)$ |
| $\Delta X_{\max , i}$ | Value of the reconstructed uncertainty of $X_{\max }$ for event number $i$ | $\left(\mathrm{~g} / \mathrm{cm}^{2}\right)$ |
| $X_{\max , i}$ | Value of $X_{\max }$ for event number $i$ | $\left(\mathrm{~g} / \mathrm{cm}^{2}\right)$ |
| $\overline{X_{\max }}$ | Average value of $X_{\max }$ of all the events | $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ |
| $R_{0, i}$ | Value of $R_{0}$ for event number $i$ | $(\mathrm{~m})$ |
| $\overline{R_{0}}$ | Average value of $R_{0}$ of all the events | $(\mathrm{m})$ |

A correlation factor of 1 represents a linear functional relation which has a positive slope and -1 represents a linear functional relation which has a negative slope. The factor 0 represents there is no linear functional relation.

The value of $\overline{X_{\max }}$ and $\overline{R_{0}}$ are calculated as follows ${ }^{[2]}$ :

$$
\begin{align*}
& \overline{X_{\max }}=\frac{\sum_{i=1}^{n} w_{i} X_{\max , i}}{\sum_{i=1}^{n} w_{i}}  \tag{20}\\
& \overline{R_{0}}=\frac{\sum_{i=1}^{n} w_{i} R_{0, i}}{\sum_{i=1}^{n} w_{i}} \tag{21}
\end{align*}
$$

The weight of an event in the correlation factor is chosen to depend on its relative uncertainty. In this way, the quality of the data is taken into account. The reconstruction algorithm calculates only statistical uncertainties of $X_{\max }$ and $R_{c}$, but there are systematic uncertainties as well ${ }^{[2,21]}$. This has to be added by hand. Additional uncertainties on $X_{\max }$ could arise due to the geometrical reconstruction, the atmospheric model and atmospheric conditions. This gives a total contribution to the uncertainty of approximately $23.6 \mathrm{~g} / \mathrm{cm}^{2}{ }^{[21]}$.
No research has been done on the systematic uncertainty of the radius of curvature. An estimation of this uncertainty is needed to assign a weight to each event, to calculate the correlation coefficient. An arbitrary value of $0.08 \cdot R_{0}$ is used ${ }^{[2]}$. This is taken to be a relative error so the weight of an event in the correlation factor is not biased by the size of $R_{0}$.

Due to the fact that $R_{0}$ depends on $E$, see figure 29 , the ( $R_{0}, X_{\max }$ ) graphs has to be plotted in energy bins. Four energy bins are chosen ${ }^{[2]}$, from 3-6 EeV, 6-12 EeV, 12-24 EeV and 24-48 EeV. Above 48 EeV , there are fewer events. The results are shown in figure 30a to d. Table 4 shows the corresponding correlation factors for the different bins and the correlation factors which G. van Aar found in his master thesis.

Table 4: Correlation factors Offline.

| Energy bin | Correlation factor | Correlation factor master thesis G. van Aar ${ }^{[2]}$ |
| :--- | :--- | :--- |
| $3-6 \mathrm{EeV}$ | -0.34 | -0.28 |
| $6-12 \mathrm{EeV}$ | -0.25 | -0.24 |
| $12-24 \mathrm{EeV}$ | -0.15 | -0.27 |
| $24-48 \mathrm{EeV}$ | -0.66 | -0.61 |


$R_{0}=p_{1} X_{\max }+p_{0}$. The horizontal line of the fit is a drawing error. Correlation factor $=-0.34$. Total number of events $=269$.
$R_{0}=p_{1} X_{\max }+p_{0}$. The horizontal line of the fit is a drawing error. Correlation factor $=-0.25$. Total number of events $=210$.


Figure 30c: $R_{0}$ as a function of $X_{\max }$ for $12<E \leq 24 \mathrm{EeV}$.
$R_{0}=p_{1} X_{\max }+p_{0}$. Correlation factor $=-0.15$. Total
number of events $=85$.

It can be concluded that the correlation becomes stronger with the additional cuts imposed on the events, except for the energy bin from 12-24 EeV. The reason for this is unknown.

### 3.1.3 Comparing $X_{\text {max }}$ from the surface and fluorescence detector

To derive $X_{\max }$ from the surface detector data the value of $R_{0}$ is calculated using equation (10) and (13). After that, $X_{\max }$ can be calculated using the equations shown in figure 30 for the corresponding energy bin, but this cannot simply be allowed. It shows that $X_{\max }$ depends on $R_{0}(E)$. Therefore, a
higher energy results in a larger value of $R_{0}$, see figure 29, which results in a smaller $X_{\text {max }}$, see figure 30. Also, a high energy particle interacts earlier but has a longer development of its shower and therefore reaches its $X_{\max }$ at a larger value, see figure 7. It can be concluded that $X_{\max }$ does not only depend on $R_{0}(E)$ but also on $E$ itself. This function is required to continue the comparison between the $X_{\max }$ from the surface and the fluorescence detector.

To start, each event which passes the surface detector and fluorescence detector cuts is plotted in a three dimensional graph with its uncertainties. The energy and the corrected radius of curvature is reconstructed by the surface detector. $X_{\max }$ is reconstructed by the fluorescence detector. The result is shown in figure 31.


Figure 31: Events which passes the surface detector and fluorescence detector cuts in a 3D plot. 591 Events.

Through these points a plane is fitted using a first polynomial in $R_{0}$ and in $\log (E)$ :

$$
\begin{equation*}
X_{\max }=\left(p_{0}+p_{1} R_{0}\right) \cdot\left(p_{2}+p_{3} \log (E)\right) \tag{22}
\end{equation*}
$$

With

$$
\begin{aligned}
& p_{0}=-76 \pm 2 \\
& p_{1}=2.120 \pm 0.112 \\
& p_{2}=-10.9 \pm 0.3 \\
& p_{3}=-1.8 \pm 0.2
\end{aligned}
$$

This plane is shown in figure 32.


Figure 32: Fitted function of $X_{\max }$ as a function of energy and the corrected radius of curvature.

Now $X_{\max }$ can be calculated from the surface detector parameters $R_{c}$ and $E$ using equation (10) and (13). The result is shown in red in figure 33 . Blue represents the reconstructed parameter $X_{\max }$ from the fluorescence detector as a function of the energy reconstructed by the surface detector. The lines for proton and iron are also plotted in the graph.


For energies above 6 EeV , which equals $\log (E)=0.8, X_{\text {max }}$ from the surface detector corresponds to $X_{\max }$ from the fluorescence detector within its uncertainties. Below 6 EeV the surface detector parameter is lower. Due to an overestimation of the correlation, it might be possible that the positions of the points becomes lower. For a definite conclusion more research is needed. At approximately 25 EeV , which equals $\log (E)=1.4$, the red graph bend to a more horizontal direction. This is measured by the fluorescence data as well and also figure 7 shows this. It might be possible that there are more iron nuclei in the cosmic rays at high energies.

### 3.2 CDAS

For the reconstruction package CDAS, the same exercise is performed. The data from 2004 until 2011 is used.

### 3.2.1 Zenith correction

In the CDAS reconstructed data not all the parameters Offline reconstructs are included or have the same name. Therefore, different cuts are required which have the same effect as the Offline cuts. Table 5 shows the new cuts on the surface detector events.

Table 5: Cuts on the CDAS reconstructed parameters measured by the surface detector.

| Standard cut ${ }^{[2]}$ | Motivation |
| :---: | :---: |
| Offline Reconstruction level $\geq 4$ | The data file of Offline which the program uses for the extra cuts, is already included with this cut. |
| ICRC05=1 | This trigger level requires 5 tanks around the station, which detect the highest signal, must be working. |
| $E>3 \mathrm{EeV}$ | The surface detectors become fully efficient above 3 EeV . |
| $\theta<60^{\circ}$ | Above this zenith angle, a different reconstruction algorithm is used, which shows some biases. |
| $N_{\text {stat }}>5$ | More than 5 stations has to detect the event. With four stations a curvature fit can be plotted. With 5 tanks, also the uncertainty can be determined. |
| $\frac{\chi_{G l o b}^{2}}{N_{\text {dof }, G l o b}}<4$ | CDAS contains only global $\chi^{2}$ and $N_{d o f}$. To get the good quality fit events this cut is required. |
| Additional cut | Motivation |
| $\left\|R_{c, \text { Offline }}-R_{c, \text { CDAS }}\right\|<5 \mathrm{~km}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $\left\|E_{\text {Offline }}-E_{C D A S}\right\|<20 \mathrm{EeV}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $\left\|\theta_{\text {Offline }}-\theta_{\text {CDAS }}\right\|<1^{\circ}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $\left\|\varphi_{\text {Offline }}-\varphi_{\text {CDAS }}\right\|<2^{\circ}$ | The differences between CDAS and Offline may not be too much. Some outliers are removed. |
| $R_{C}>0$ | There is one event which has an $R_{c}<0$. This event is not reconstructed correct and therefore removed. |

The uncertainties are calculated on the same way as the uncertainties of Offline. The resulting plots of $\left(R_{c}, \theta\right)$ are shown in figure 34 a to j .


Figure 34a: Radius of curvature as a function of the zenith angle for an energy between 3 and 4.25 EeV. 8802 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$



Figure 34 c : Radius of curvature as a function of the zenith angle for an energy between 6.05 and 8.59 EeV .5425 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$



Figure 34e: Radius of curvature as a function of the zenith
angle for an energy between 12.20 and 17.32 EeV . 2105 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1(\theta)}}}
$$



Figure 34b: Radius of curvature as a function of the zenith angle for an energy between 4.25 and 6.05 EeV . 7430 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$



Figure 34d: Radius of curvature as a function of the zenith angle
for an energy between 8.59 and 12.20 EeV .3588 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos p_{1(\theta)}}
$$



Figure 34f: Radius of curvature as a function of the zenith angle for an energy between 17.32 and 24.60 EeV .1170 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1(\theta)}}} .
$$



Figure 34 g : Radius of curvature as a function of the zenith angle for an energy between 24.60 and 34.92 EeV . 529 Events. The red curve is described by:
$R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}$.


Figure 34 i : Radius of curvature as a function of the zenith angle for an energy between 49.59 and 70.42 EeV. 91 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos p_{1}(\theta)}
$$



Figure 34 h : Radius of curvature as a function of the zenith angle for an energy between 34.92 and 49.59 EeV . 222 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos p_{1(\theta)}}
$$



Figure 34 j : Radius of curvature as a function of the zenith angle for an energy between 70.42 and 100 EeV. 27 Events. The red curve is described by:

$$
R_{c}=\frac{p_{0}}{\cos ^{p_{1}(\theta)}}
$$

A plot of $\alpha$ as a function of $\log (E)$ results in figure 35 .


Figure 35: Fit results of the power $\alpha$ as a function of $E . \alpha=p_{2}(\log (E))^{2}+p_{1} \log (E)+p_{0}$
The function $\alpha(E)$ becomes:

$$
\begin{equation*}
\alpha(E)=(-0.11 \pm 0.07) \cdot(\log (E))^{2}+(0.12 \pm 0.13) \log (E)+(1.36 \pm 0.07) \tag{23}
\end{equation*}
$$

Without the additional cuts, the function $\alpha(E)$ is ${ }^{[2]}$ :

$$
\begin{equation*}
\alpha(E)=(-0.11 \pm 0.05) \cdot(\log (E))^{2}+(0.11 \pm 0.11) \log (E)+(1.37 \pm 0.06) \tag{24}
\end{equation*}
$$

It can be concluded that equation (23) equals equation (24) within its uncertainties.

Figure 36 shows that $\chi^{2} / N_{\text {dof }}$ of all the fits in figure 34 are below 1 . This means that the uncertainties are not underestimated.


For all the events reconstructed by CDAS, $R_{0}$ is calculated using equation (10) and the derived function for $\alpha$, equation (23). The results are shown in figure 37. Also here the corrected radius of curvature is larger when the energy is higher.

### 3.2.2 Correlation between $X_{\max }$ and $\boldsymbol{R}_{\mathbf{0}}$

Due to the fact that CDAS does not contain fluorescence data, $X_{\text {max }}$ reconstructed by Offline is used in the same way as in chapter 3.1. The $\left(R_{0}, X_{\max }\right)$ graphs are plotted in the same energy bins as chosen before. The results are shown in figure 38a until d. Table 6 shows the corresponding correlation factors for the different bins.

Table 6: Correlation factors CDAS.

| Energy bin | Correlation factor |
| :--- | :--- |
| $3-6 \mathrm{EeV}$ | -0.31 |
| $6-12 \mathrm{EeV}$ | -0.32 |
| $12-24 \mathrm{EeV}$ | -0.33 |
| $24-48 \mathrm{EeV}$ | -0.56 |



Figure 38a: $R_{0}$ as a function of $X_{\max }$ for $3<E \leq 6 \mathrm{EeV}$.
$R_{0}=p_{1} X_{\max }+p_{0}$. The horizontal line of the fit is a drawing error. Correlation factor $=-0.31$. Total number of events $=338$.


Figure 38 b : $R_{0}$ as a function of $X_{\max }$ for $6<E \leq 12 \mathrm{EeV}$. $R_{0}=p_{1} X_{\max }+p_{0}$. The horizontal line of the fit is a drawing error. Correlation factor $=-0.32$. Total number of events $=237$.


Figure 38c: $R_{0}$ as a function of $X_{\max }$ for $12<E \leq 24 \mathrm{EeV}$.
$R_{0}=p_{1} X_{\max }+p_{0}$. Correlation factor $=-0.33$. Total number of events $=98$.


Figure 38d: $R_{0}$ as a function of $X_{\max }$ for $24<E \leq 48 \mathrm{EeV}$.
$R_{0}=p_{1} X_{\max }+p_{0}$. Correlation factor $=-0.56$.
Total number of events $=27$.

It can be concluded that there is a correlation between $R_{0}$ and $X_{\max }$. Table 7 shows all the correlation coefficients again.

Table 7: The results of the correlation coefficients.

|  | Correlation coefficient <br> Offline |  | CDAS |
| :---: | :---: | :---: | :---: |
| Energy bin | With additional cuts | Master thesis G. van Aar ${ }^{[2]}$ | With additional cuts |
| $3-6 \mathrm{EeV}$ | -0.34 | -0.28 | -0.31 |
| $6-12 \mathrm{EeV}$ | -0.25 | -0.24 | -0.32 |
| $12-24 \mathrm{EeV}$ | -0.15 | -0.27 | -0.33 |
| $24-48 \mathrm{EeV}$ | -0.66 | -0.61 | -0.56 |

In general the packages show good agreement, with the exception of the 12-24 EeV energy bin. Here the Offline package shows an outlier. Table 8 shows the relations between $R_{0}$ and $X_{\max }$ for Offline and CDAS.

Table 8: The results of $R_{0}$ as a function of $X_{\max }$.

| Energy bin <br> (EeV) | Offline <br> $\boldsymbol{R}_{\mathbf{0}}(\mathbf{k m})$ | CDAS <br> $\left.\boldsymbol{R}_{\mathbf{0}} \mathbf{( k m}\right)$ |
| :---: | :---: | :---: |
| $3-6$ | $(-0.024 \pm 0.003) X_{\max }+(25 \pm 2)$ | $(-0.016 \pm 0.002) X_{\max }+(19 \pm 2)$ |
| $6-12$ | $(-0.060 \pm 0.010) X_{\max }+(53 \pm 8)$ | $(-0.033 \pm 0.005) X_{\max }+(33 \pm 4)$ |
| $12-24$ | $(-0.016 \pm 0.006) X_{\max }+(21 \pm 5)$ | $(-0.014 \pm 0.003) X_{\max }+(19 \pm 3)$ |
| $24-48$ | $(-0.07 \pm 0.03) X_{\max }+(6 \pm 3) \cdot 10^{1}$ | $(-0.0353 \pm 0.0105) X_{\max }+(36 \pm 9)$ |

If $R_{0}$ is calculated for one event with the equations in table 8 , a different value is found for CDAS and Offline, see figure 39.


Figure 39: Corrected radius of curvature as a function of the energy

When a primary particle has an higher energy, it will interacts earlier in the atmosphere and therefore $R_{0}$ becomes bigger. This last is an indication that the radius of curvature has a correlation between the height of the first interaction. This can be investigated in the future.

### 3.2.3 Comparing $X_{\text {max }}$ from the surface and fluorescence detector

To derive $X_{\max }$ from the surface detector data the value of $R_{0}$ is calculated using equation (10) and (23). Also here $X_{\max }$ cannot be calculated from only the binned information. There has to be an equation for $X_{\max }$ as a function of $E$ and $R_{0}(E)$.

Each event which passes the surface detector cuts for CDAS and the fluorescence detector cuts for Offline is plotted in a three dimensional graph with its uncertainties. The energy and the corrected radius of curvature is used from the CDAS surface detector data. $X_{\max }$ is the Offline reconstructed parameter from the fluorescence detector. The result is shown in figure 40.


Figure 40: Events which passes the CDAS surface detector and Offline fluorescence detector cuts in a 3D plot. 706 Events.

Through these points a plane is fitted with a first polynomial in $R_{0}$ and in $\log (E)$ :

$$
\begin{equation*}
X_{\max }=\left(p_{0}+p_{1} R_{0}\right) \cdot\left(p_{2}+p_{3} \log (E)\right) \tag{25}
\end{equation*}
$$

With

$$
\begin{aligned}
& p_{0}=-83 \pm 7 \\
& p_{1}=3.7 \pm 0.4 \\
& p_{2}=-11 \pm 1.0 \\
& p_{3}=-3.1 \pm 0.5
\end{aligned}
$$

This is shown in figure 41. The plane fitted for Offline is represented here as a transparence plane.


Figure 41: Fitted function of $X_{\max }$ as a function of energy and the corrected radius of curvature. The transparence plane represents the fitted plane for Offline.

The plane determined from the Offline data differs from the plane from the CDAS data. It is calculated more horizontal.

Now $X_{\max }$ can be calculated from the surface detector parameters $R_{c}$ and $E$ reconstructed by CDAS. The result is shown in red in figure 42. Blue represents the Offline reconstructed parameter $X_{\max }$ from the fluorescence detector as a function of the energy of the surface detector reconstructed by CDAS. The lines for proton and iron are also plotted in the graph.
$X_{\text {max }}$ from the surface (CDAS) (red) and fluorescence detector (Offline) (blue)


Figure 42: $X_{\max }$ calculated from $R_{c}$ reconstructed by CDAS from the surface detector (red) and
$X_{\max }$ measured by the fluorescence detector reconstructed by Offline (blue). The statistics of "pXEdataFD" is for the fluorescence data, "pXEcalculatedSD" for the surface detector data. The orange lines represent the proton lines. The green lines represent the iron lines.

For energies above 4 EeV , which equals $\log (E)=0.6, X_{\text {max }}$ from the surface detector corresponds to $X_{\max }$ from the fluorescence detector within its uncertainties. Below 4 EeV the surface detector parameter is lower. The reason for this might be again an overestimated correlation.
At approximately 13 EeV , which equals $\log (E)=1.1$, the red graph bend also to a more horizontal direction. This value is smaller than for Offline. The reason for this is not completely clear. More research is needed.

## 4. Conclusion

The VEM signals for Offline and CDAS are different, mainly due to a different charge calibration but also to a different correction for the saturated photomultipliers, a different baseline and a different start time. Therefore, the signal at 1000 m from the shower axis is different too. The median difference between Offline and CDAS for air showers with an energy greater than 30 EeV and a zenith angle smaller than $60^{\circ}$ is: $(-10.4 \pm 0.5)$ VEM. The reconstructed energy, which is a result of this value, is therefore also different: $(-2.84 \pm 0.12) \mathrm{EeV}$. For the radius of curvature, the zenith and azimuth angle the median differences becomes respectively: $(0.00 \pm 0.04) \mathrm{km},(0.069 \pm 0.007)$ deg and $(-0.032 \pm 0.016)$ deg.

The corrected radius of curvature is correlated with $X_{\max }$. The correlation factors for the Offline package with an energy of the primary between 3 and 6 EeV is -0.34 . For 6-12 EeV: -0.25. For 12-24 $\mathrm{EeV}:-0.15$ and for $24-48 \mathrm{EeV}:-0.66$. For CDAS these factors become respectively: $-0.31,-0.32,-0.33$ and -0.56 . More research is needed to the differences between the packages to explain this.

In general a smaller $X_{\max }$ results in a bigger zenith corrected radius of curvature, $R_{0}$. $R_{0}$ also increases as a function of energy for CDAS and Offline. When a primary particle has an higher energy, it will interact earlier in the atmosphere and therefore $R_{0}$ becomes bigger. This last is an indication that the radius of curvature correlates with the height of the first interaction. This can be investigated in the future.

The reconstructed value of the depth of the shower maximum from the fluorescence detector is compared to the calculated parameter using the radius of curvature from the surface detector. For energies above 6 EeV for Offline and 4 EeV for CDAS the calculated parameter is equal to the reconstructed parameter within uncertainties. Below these values the calculated parameter is lower. A reason for this can be an overestimation of the correlation, which results in lower points. At approximately 25 EeV for Offline and 13 EeV for CDAS the calculated parameter does not rise with energy as fast as before this border. It is possible that only heavy nuclei arrive at Earth at high energy, because for those depth of the shower maximum is smaller. For a well defined conclusion, more research is needed.

Figure 43 shows the calculated $X_{\max }$ using $R_{c}$ from the surface detector reconstructed by Offline and CDAS and the measured point according to figure 7.


Figure 43: The blue and red points represents the points calculated using $R_{c}$ from the surface detector reconstructed by CDAS and Offline. The pink points are the measured data with the fluorescence detector from figure 7.

Only the last three energy bins of the measured $X_{\max }$ are equal to the calculated $X_{\max }$. A reason for this is unknown. The points for CDAS increases faster with energy than Offline. At high energies CDAS flattens out earlier than Offline, but it seems to be flattens to the same value of $X_{\max }$. It can be concluded that the type of reconstruction package has an influence on the physics and that, using the calculated $X_{\max }$, the upper reach of measured energies is increased with a factor 2.

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## VAppendices

## Appendix I: Derivation of the shower maximum

The value of $X_{\max }$ is proportional to the logarithm of the energy of the primary particle ${ }^{[22]}$ :

$$
\begin{equation*}
X_{\max } \propto \ln (E) \tag{26}
\end{equation*}
$$

Where:

$$
\begin{array}{lll}
X_{\max } & \text { Depth at which the lateral distribution of charged particles is at its maximum } & \left(\mathrm{g} / \mathrm{cm}^{2}\right) \\
E & \text { Energy of the primary particle } & (\mathrm{eV})
\end{array}
$$

The proportionality coefficient depends on the nature of hadronic interactions, most notably on the multiplicity, elasticity and cross section of ultra high energy collisions of hadrons with air ${ }^{[22]}$.

Suppose that the primary particle has an unit of energy. If there are two particles in which this primary decays, think for instance about pair production, the energy per particle is on average the half of the energy of the parent. A toy model give the main characteristics of an air shower. Because an air shower mainly contains electromagnetic particles, pair production and bremsstrahlung are very frequent, thus this model provides similar characteristics as a real air shower. The decay continues for many generations, see figure 44.


Figure 44: A schematic view of the air shower. The primary particle (the biggest point) and its secondary particles (the small points).

Generally:

$$
E_{\text {part }}=\frac{E}{2^{n}}
$$

Where:

$$
\begin{array}{lll}
E_{\text {part }} & \text { Energy per secondary particle } & (\mathrm{eV}) \\
n & \text { Generation } & (-)
\end{array}
$$

There is a critical energy after which no new particles are created. If this is called, $E_{c}$, the number of decays equals:

$$
n=\frac{\log \left(\frac{E}{E_{c}}\right)}{\log (2)}
$$

Where:
$E_{C} \quad$ Critical energy at which no new particles are created

The value of $X_{\max }$ is proportional to the generation, $n$. This justifies equation (26).

If the cosmic ray is not one particle, but a combination of several particles, like a nucleus, the model in figure 44 can be used too. Every particle in the nucleus can be seen as a primary particle in figure 44. The energy of this particle becomes:

$$
E=\frac{E_{n u c l}}{A}
$$

Where:

$$
\begin{array}{lll}
E_{\text {nucl }} & \text { Energy of the nucleus } & (\mathrm{eV}) \\
A & \text { Mass number of the nucleus } & (-)
\end{array}
$$

Therefore, $X_{\max }$ is proportional to ${ }^{[13]}$ :

$$
\begin{equation*}
X_{\max } \propto \ln \left(\frac{E_{n u c l}}{A}\right) \tag{27}
\end{equation*}
$$

## Appendix II: Air showers with big differences between two reconstructions

This appendix shows why some parameters reconstructed by the reconstruction package CDAS are different from the parameters reconstructed by the reconstruction package Offline for the same air shower, called an event. The events are out layers of the following cuts:
$\left|R_{c, \text { Offline }}-R_{c, \text { CDAS }}\right|<5 \mathrm{~km} ;$
$\left|E_{\text {Offline }}-E_{C D A S}\right|<20 \mathrm{EeV}$;

A lot of the out layer events contains saturated photomultipliers in one or more stations, see figure 45 a until c. Therefore, the lateral distribution fit is not correct, see figure 46 . This results in an energy which is not reconstructed correctly.

## Photomultiplier 1



Figure 45a: Example of a saturated signal of photomultiplier 1 in a station. The Auger ID number of this event is 20103803677.

Photomultiplier 3


Figure 45c: Example of a saturated signal of photomultiplier 3 in a station. The Auger ID number of this event is 20103803677.

## Photomultiplier 2



Figure 45b: Example of a saturated signal of photomultiplier 2 in a station. The Auger ID number of this event is 20103803677.

## Lateral distribution fit



Figure 46: Lateral distribution fit reconstructed by CDAS with the station which contains the three saturated photomultipliers shown in figure 45 a until c. The Auger ID number of this event is 20103803677.

The values of the measured signals in the fit reconstructed by CDAS do not correspond to the values represented in the list that shows the measured signals. Figure 47 shows a print screen of the event where this happens. The reason for this is unknown.


Figure 47: Event reconstructed by CDAS which shows a different in the signal in the list (the top center) and the graph (the top right). Auger ID: 200918800775.

There are also events where the points in the curvature fit contain a very small uncertainty. See figure 48 . The curvature fit will be attracted to this points. This results in a curvature which is not correct.

Curvature fit

$r$ (m)
Figure 48: Curvature fit with point with small uncertainties, which influenced the curvature fit. Auger ID: 200917001567. Reconstructed by Offline.

A T5 of 0 means that not all the stations around the station with the highest signal are present and functioning at the time. It is possible that the shower core is positioned outside the array and therefore the air shower is not reconstructed well. This is shown in figure 49.


Figure 49: Shower core (red) positioned outside the array. The green stations are the stations which detected a signal. The blue points are the stations with no signal. The triangle shows the three highest signal stations.

## Appendix III: Original job description (Dutch)

Tijdsduur: 17 weken (6/2/2012-1/6/2012)
Locatie: Afdeling experimentele hoge energie fysica, Radboud universiteit Nijmegen
Onderzoeksvraag: Is het mogelijk deeltjes-detectoren op aarde te gebruiken om de aard van kosmische straling te onderzoeken?

Als een hoog energetisch deeltje uit de ruimte in de aardatmosfeer doordringt, ontstaat er een interactie tussen dit deeltje en de moleculen in de atmosfeer. Dit heeft tot gevolg dat er een lawine van secundaire deeltjes ontstaat die met de lichtsnelheid richting aarde bewegen. De hoogte waarop een lawine ontstaat heeft te maken met de aard en energie van het kosmische deeltje. In het algemeen geldt dat een samengesteld deeltje eerder reageert, en dat dus de deeltjeslawine hoger in de atmosfeer ontstaat dan dat van een waterstofkern. Ook geldt dat als de energie hoger is, de eerste reactie eerder plaatsvindt. In dat geval ontwikkelt de lawine zich langer, zodat de hoogte waar het aantal deeltjes maximaal is juist dieper in de atmosfeer ligt.
Deze deeltjeslawines zijn waar te nemen door het fluorescentielicht dat in de atmosfeer ontstaat, maar ook met deeltjesdetectoren op de Aarde. Die deeltjesdetectoren meten de aankomsttijden en intensiteit van een deeltjesfront voor ieder deeltje dat uit de ruimte komt. Beide technieken worden gebruikt bij het Pierre Auger observatorium in Argentinie, waar onze gegevens worden verzameld. We gaan proberen om de kromtestraal van het deeltjesfront te interpreteren als een maat voor de hoogte van de eerste interactie.
De eerste studies van een master student en een bachelorstudente, beide van de RU, hebben aangetoond dat deze kromtestraal een correlatie vertoont met de hoogte waarop de meeste deeltjes in de lawine zitten (shower-max). Die laatste informatie komt uit de fluorescentiedetectie. In het algemeen geldt dat voor een hogere shower-max de kromtestraal op aarde groter is. Ook geldt dat bij een hogere energie van het inkomende deeltje de kromtestraal groter is. Dit laatste is een indicatie dat de kromtestraal iets te maken kan hebben met de hoogte van de eerste interactie. De vraag is of deze interpretatie juist is.

## Werkzaamheden

De stageperiode wordt grofweg verdeeld in 4 periodes van 4 weken. De indeling is min of meer als volgt:
Periode 1:
Introductie over het Auger experiment en air shower ontwikkeling. Daarnaast het leren omgaan met de programmatuur (root, event display)
Periode 2:
Herhaling van de laatste resultaten uit de bachelorstage van Marie Lanfermann. Dit geeft meer inzicht in de gebruikte variabelen en geeft een eerste ijkpunt voor de analyse
Periode 3:
Herleiding van de kromtestraal tot een hoeveelheid atmosfeer boven de eerste interactie.
Vergelijking hiervan met de locatie van shower-max en energie. Vergelijk deze waarde met theoretische botsingsdoorsnedes, is er een relatie?
Periode 4:
Analyseren van data en verslaglegging (laatste in het Engels, waarschijnlijk moeten hier minimaal 3 weken voor uitgetrokken worden)
De stage vindt plaats in een actieve onderzoeksgroep waarin mensen regelmatig reizen. Dat betekent dat de begeleiding niet continu door dezelfde persoon gegeven wordt. Verantwoordelijk voor de begeleiding is dr. C. Timmermans. In de praktijk nemen de 3 promovendi in de groep een groot deel van deze taken op zich. Het ligt niet in de lijn der verwachting dat alle 4 de personen voor langere tijd afwezig zijn, waardoor er altijd een expert aanwezig is.


[^0]:    Figure 5: Schematic view of the fluorescence telescope ${ }^{[10]}$.

