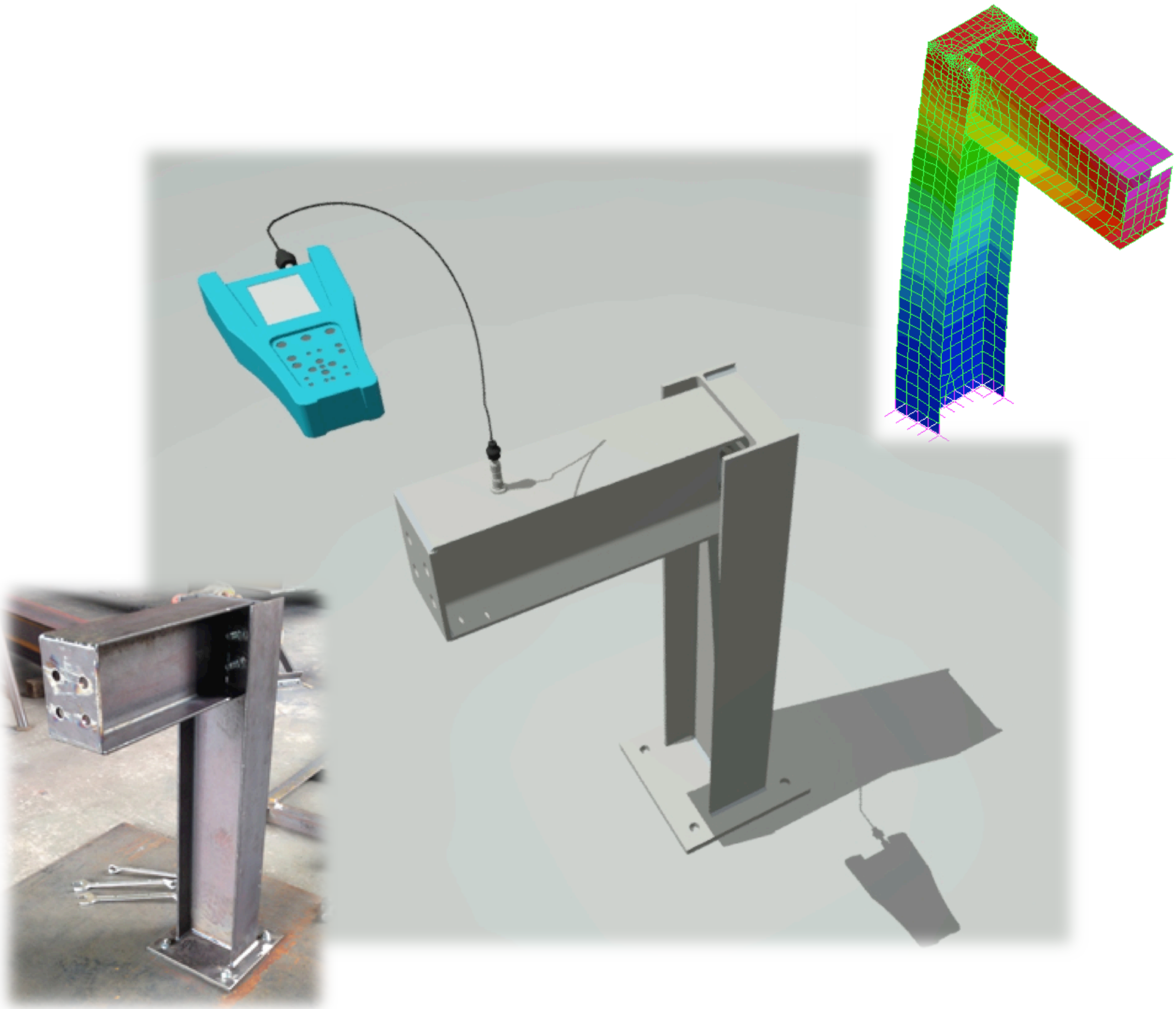
PBA Projects

FEA Research on Bolted Joints



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Practical Finite Element Analysis
Research on bolted joints for the mining industry

Graduation project on Mechanical Engineering

PBA Projects – 09094881 – Marinus van Wijk

Abstract

Finite element analysis on large structures in the mining industry is mainly set up with beam-elements. During the modelling of the structures a couple of assumptions are made to ensure that the structure is not too complex. The connections between the beams are usually released in a couple of direction to simulate the bolted connection used.

However, questions remain regarding the accuracy of the assumption to end-release the beams. The purpose of this study is to compare the accuracy of the Finite Element Analysis (FEA) models against measurements on a fabricated structure. For the fabricated structure a simple column was used with a beam connected to the top of the column with an endplate and four bolts. The FEA models were set up in beam- and plate-elements and calculated the same natural frequencies.

The measurements on the fabricated structure returned lower frequencies. Adjusting the FEA-models improved the accuracy of the calculated natural frequency. The difference between the measured and the calculated natural frequencies is due to the way the structure was anchored to a concrete slab and the gusset used on the column.

This concludes the assumption that fixed nodes have a high impact on the natural frequency and makes the simple structure used too complex. Plate-element models give higher compared to beam-element models and can be used to determine the correct natural frequency. However, plate-element models take significantly more time to model and calculate. Furthermore it is now known that the FEA calculations are stiffer than structure.

Advice given to PBA Projects is to continue the research on a simply supported beam. The beam should not be too long to keep the influence of the vibrations of the baseplates on the beam to minimum.

Keywords: Finite element analysis, natural frequency, bolted connections

Preface

This thesis has been written by me as graduate at PBA Projects where I've been working as a structural engineer.

The research project was set up by me as intern and approved by Che Smit. During the project Che Smit guided me as intern by giving new insights to the research.

This report gives a clear overview of the research done on bolted joints in I-beams with Finite element analysis and measurements.

The report is intended for the Examination Board at The Hague University of Applied Sciences and for Che Smit at PBA Projects.

Special thanks to PBA Projects for giving me the opportunity to do an internship abroad with a challenging assignment, especially Che Smit for his positive contribution. Further more I'd like to thank Nathan at PBA Engineering for the opportunity and the guidance to fabricate the structure for my research and the possibility to do the needed tests at the factory. I'd also like to thank mr. Erdurcan for the guidance given from the university.

Marinus van Wijk
Cape Town, December 2014

Table of Contents

Glossary & Symbol list

Definition

Damping	:	Dissipation or absorption of energy. Damping is usually assumed to be “vicious damping” which means that it is proportional to the velocity.
Frequency	:	This is the rate at which a harmonic quantity varies with time. It is important to distinguish between a cyclical frequency, f, which is measured in cycles/second, and radial frequency, ω, which is measured in radians/second. These are directly related by: $\omega = 2\pi f$
Harmonic	:	Any quantity that varies with time according to: $Q = Q_0 \sin(\omega t + \phi)$. Many of the loads applied by industrial equipment are harmonic loads.
Model	:	A model is a representation of a complex object. In dynamic design a model typically refers to a computer representation of a real structure.
Mode shape	:	A natural shape in which a structure will vibrate. This is mathematically referred to as an “eigenvector”.
Natural Frequency	:	The frequency at which a structure will naturally vibrate in the absence of any applied force.

Abbreviations

FFT	:	Fast Fourier Transform
FEA	:	Finite Element Analysis
FEM	:	Finite Element Modelling
DFT	:	Discrete Fourier Transform
CAD	:	Computer Aided Design

Symbols

$\ddot{\theta}$	:	Angular acceleration (rad/s^2)
θ	:	Angular position (rad)
f	:	Frequency (Hz)
θ_0	:	Initial angular position (rad)
$\dot{\theta}_0$	:	Initial angular velocity (rad/s^2)
x_0	:	Initial position (m)
$\dot{x}_0$	:	Initial velocity (m/s)
m	:	Mass (kg)
ω_n	:	Natural circular frequency (rad/s)
f_n	:	Natural frequency (Hz)
ϕ	:	Phase angle (rad)
k	:	Rigidity (Nm/rad)
t	:	Time (s)

List of Charts

List of Figures

List of Tables

1 Introduction

Mechanical systems assembled from metallic components will naturally vibrate at a frequency when any external applied force is applied. This frequency is called the natural frequency and is of importance to the designer of a system. In order to save time in the design process mechanical systems are often modelled in FEA packages to calculate the deflection at certain frequencies and compare to the allowable deflection.

For numerical simulations of the dynamical behaviour of structures, the finite element method (FEM) has established itself as a standard tool. Even though the mass and the stiffness distribution of a structure can be modelled within FEA quite precisely, the fixed connections in FEA are infinite stiff compared to the connection in real life. Thus the results need to be confirmed by performing measurements on the model.

Structures fabricated at PBA Projects are mainly built out of I-beams. The structures are used for the mining industry, supporting many machines like shakers, mills, pumps, etc. The I-beams of the structures are connected to each other with a welded endplate to an end of a beam and holes in the other beam. Usually four M20 bolts are used for the connection.

The structures are modelled in a FEM package, Strand7, and tested in 180 cases that simulate the machines installed but also heavy storm winds and earthquakes. (Figure 1) The connections between the beams are modelled in a specific way, by releasing the ends of the beam in natural frequency analysis.

The main goal of this thesis is to research the bolted connections between two I-beams at PBA Projects to confirm the results found by the FEA package used. Not only is it needed to confirm the results but also the way beam connections are modelled needs to be confirmed.

Therefore a small structure will be designed, modelled in FEA and CAD, fabricated, and measurements done. In the whole process from designing till the measurements many tasks that a mechanical engineer learn during his or her study will be used to make finish this project successfully.

At the end of the project the goal is to identify what connection in FEA is to be used for the bolted connection.

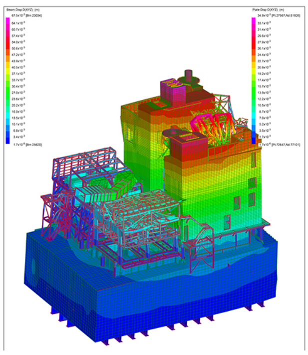


Figure 1 FEA analysis done by PBA Projects on a mining plant

2 Background

The project was set up at PBA Projects due to the need to approve the results that are calculated in the FEA. The research was mainly done at the office of PBA Projects in Belville, Cape Town. Fabrication and measurements on the structure was done at PBA Engineering at Parow, Cape Town.

2.1 PBA Projects

PBA Projects is an Engineering, Procurement and Construction management company based in Cape Town, South Africa whose aim is to provide professional multi-disciplinary engineering solutions to their clients. PBA mainly advise, design and construct processing plants for the mining industry. PBA Projects is the lead company in the PBA group.

PBA develops and constructs modular processing plants for the processing of raw ore in order to filter and process materials like diamonds, gold, coal and other essential materials. These structures undergo high cyclic and vibratory stress in harsh conditions.

2.2 PBA Engineering

PBA Engineering is a manufacturing company that specialises in the manufacturing of modular process plants and heavy engineering solutions for the mining, marine and general engineering industries. PBA Engineering manufactures the plants for PBA Projects but also does manufacturing work for other companies like Damen Shipyards Cape Town.

The model was fabricated at PBA Engineering as my graduation project, under the guidance of colleagues at PBA Engineering.

Figure 2 MODCO, a plant designed by PBA Projects and manufactured by PBA Engineering

3 Research

3.1 Vibration analysis

The physical systems we know can all vibrate. All the systems have frequencies at which vibration naturally occurs and the modal shapes that they assume are properties of the system. These frequencies can be determined analytically using FEA Modal Analysis.

Analysis of vibration modes is often overlooked, but is a critical component of a design. Structural elements such as complex steel floor systems in buildings can be particularly prone to noticeable vibration, irritating building occupants or disturbing sensitive equipment. Inherent vibration modes in mechanical supports or structural components can shorten the systems life and cause unanticipated premature or complete failure. The failure often results in hazardous situations. To assess the potential failure or damage resulting from the rapid stress cycles of vibration a detailed fatigue analysis.

With detailed modal analysis the fundamental vibration mode shapes and corresponding frequencies can be determined. For basic components of a simple system it can be relatively simple. When qualifying a complex mechanical device or a complicated stricter exposed to periodic wind loading this on the other hard will be extremely complicated. Both systems require accurate determination of natural frequencies and mode shapes using techniques such as Finite Element Analysis.

3.1.1 Natural frequency

The building block of all dynamic analyses is the modal analysis, which reports the natural frequencies and corresponding principal mode shapes of the system under evaluation. In other words, when performing a modal analysis, you solve for the distinct deformation shapes that the vibrating system will assume at each of its preferred oscillating frequencies. These concepts are better presented with the aid of a simple example. (Figure 3)

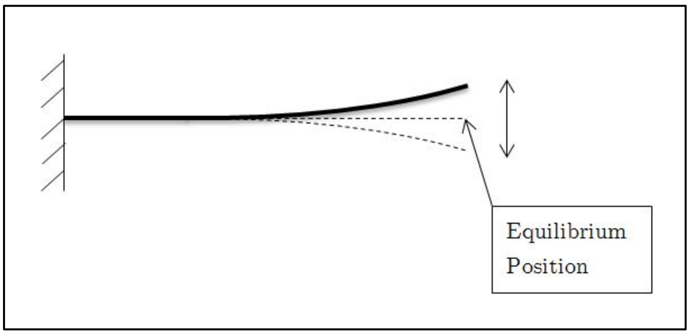


Figure 3 Free vibration of a cantilever beam, first mode.

Referring to a cantilever beam, it is intuitive that a thin beam fixed at one end will vibrate or fluctuate most easily about its fixed point with no additional "nodes" or bends (inflection points) in its deformed shape. The natural frequency (ω_n) corresponding to this mode shape is essentially the oscillatory speed with which the beam moves from one extreme to the other and back. This speed is defined by two fundamental physical parameters of the beam: *mass* (m) and *rigidity* (k) or "spring-back."

$$\boldsymbol{\omega}_n \propto \sqrt{\frac{k}{m}}$$

Equation 1

The mass contribution to this equation is understood by considering inertia. The more mass (inertia) that the beam has, the harder it is for the beam to change directions when fluctuating, and consequently, the slower the motion. Spring-back is the force that resists the displacement of the beam from its equilibrium position. When the beam is bent past this position and then released, its material elasticity tries to snap it back into place. Inertial effects prevent the beam from immediately returning to a spring-back condition on the other side, and the cycle begins again. The more rigidity, the faster this happens.

The interactions of these two parameters balance out to provide a constant oscillation speed, which is the first natural frequency of the system. This first natural is the lowest speed at which the beam will vibrate after all external excitations are removed, a state known as free vibration and governed by the following equation:

$$\ddot{\theta} + \omega_n{}^2\theta = 0$$

Equation 2

Where $\ddot{\theta}$ is the angular acceleration of the beam and θ is its angular position away from equilibrium. The solution to this equation will give the mode shape corresponding to the natural frequency. Because oscillatory motion is expected, the solution type can be assumed to be of the form

$$\theta = C\sin(\omega_n t + \psi)$$

Equation 3

Where C and ψ are constants determined by the initial conditions of the system. Letting θ_0 and $\dot{\theta}_0$ be the initial position and velocity of the beam, the equation becomes

$$\theta = \sqrt{\theta_0{}^2 + \left(\frac{\theta_0}{\omega_n}\right)^2}\sin\left[\omega_n t + \tan^{-1}\left(\frac{x_0\omega_n}{\dot{x}_0}\right)\right]$$

Equation 4

And it describes the first oscillatory mode of the beam.

Note that the ω_n units are radians per unit time. It is often more convenient to describe this natural frequency in terms of cycles per unit time (cycles per second is common) using the following variable.

$$f_n = \frac{\omega_n}{2\pi}$$

Equation 5

3.2 Finite Element Analysis

Finite Element Analysis (FEA) is a tool for evaluation of structures or systems by providing an accurate prediction of a component's response that is subjected to thermal and structural loads. Mechanical steady or cyclic loads or thermal loads can all be included in structural analyses. For thermal analyses convection, conduction and radiation heat transfer, but also various thermal transients and thermal shocks can be included.

FEA is often used for the verifying of design integrity and identifying of critical location on components without having to build the part or assembly. By modelling the structure into thousands of small pieces, called finite elements, the program can analyse the model. The breaking down of the entire structure into small elements is called discretization. For each element the solution to the governing equations is closely approximated, resulting in a number of equations for every element that needs to be solved. However, each element interacts with its neighbours, where each elements response strongly depends on that of its neighbours and so on.

Therefore, the element equations are not solvable by rendering the solution over each element. Instead, the equations from all the elements over the entire structure need to be solved simultaneously. To perform equations simultaneously is a task that can only be performed by computers. It is noteworthy that if a structure is broken down into smaller elements, and therefor a larger number of elements, the number of simultaneous equations that needs to be solved will increase. Typically, to calculate more complex structures more computer power is needed. To save time, finer meshes are only used in the locations where the highest stress or heat flow may exist. This will allow quicker solutions.

FEA was largely developed by aerospace engineers in the 1950's to design aircraft structures. The method has continually developed, also thanks to the rapid growth of computing power, and is now a tool for many technical analyses.

- Very accurate tool for failure analysis purposes
- Distinguish between failures due to design deficiencies, fabrication errors, materials defects and abusive use.
- Quantify design defects, buckling, fatigue and code compliance
- Provides excellent visual aids and animations for deformations

FEA will be used in this thesis for calculating the natural frequency of structures.

3.2.1 Natural Frequency solver in Finite Element Analysis

The natural frequency is solved in Strand7 by calculating the free vibration frequencies and corresponding vibration modes of an undamped structure. The natural frequency analysis is calculated with the following eigenvalue formula:

$$[K]\{x\} = \omega^2[M]\{x\}$$

Equation 6

with;

$[K]$ Global stiffness matrix

$[M]$ Global mass matrix

$\{x\}$ Vibration mode vector

ω Natural (circular) frequency (rad/sec)

The solver gets its results by performing the following steps:

- Calculation and assembly of the element stiffness and mass matrices is done to form the global stiffness and mass matrices. When calculating the stiffness, the material temperature dependency is taking into consideration through the, by the user nominated, temperature case. The solver gives the option to use wither a consistent or lumped mass matrix. The geometric stiffness matrix is formed and assembled to the global stiffness matrix when an initial solution is applied. In this process the constraints are assembled.
  The solver is a linear solver therefor all nonlinear attributes that has effect on the stiffness will be based on the current material status and geometry. For nonlinear elastic material the current material modulus values will be used and for plastic material the initial modulus. If geometric nonlinearity is considered the current geometry is used.
- The solver modifies the stiffness matrix if a shift values is applied. To determine the modes near the desired value the shift may be used.
- To get frequencies and the corresponding mode shapes the solver solves the eigenvalue problem using the Sub-Space Iteration Method.

4 Determination of Model

4.1 Description of the model

To determine the natural frequencies of the bolted joint connection a model of the connection is needed. The model is defined by modelling two 203x133x25 I-beams (SA: I Sections (Parallel Flange) - 203x133x25), the one functioning as a 1m column and the other as a 0.5m beam with an endplate welded to one of its ends.

The main reason for choosing the 203x133x25 I-beams is that PBA uses these beams on many of their structures. The first layout of the model can be found in Figure 4 below. The beam will be fixed to the web and flange of the column, while the beam will be tested over the inner yy-axis.

The joint due to the endplate have an effect on the natural frequency. A full endplate has greater stiffness since the plate is welded to the I-beam's flanges while a half endplate is only welded to the web of the I-beam. To be able to determine the difference between the two endplates and save costs, both ends of the I-beam will have a different endplate. The extra endplate at the end of the I-beam will have an effect on the natural frequency, since it will add mass. However it should be minimal in the force-direction that we are interested in therefore the choice of a cost-reduced method is justified.

Figure 4 Inventor model of the structure, attached to the half endplate

When measuring the natural frequency on the fabricated model, a force will be applied on the outer end of the beam in the –Y-direction (using Figure 4's origin). This will result in the rotation about the Z-axis. Therefore we are only interested in the natural frequency modes that result in a rotation about the Z-axis.

4.2 Inventor model

The Inventor model was created after it was determined how the model should be tested on natural frequencies. The model is needed to make manufacturing drawings of the model. See Figure 5.

The model is build up from two 203x133x25 I-beams. The first is 1000mm and will function as a column. The column is welded to a baseplate of 300x300x20mm and at the top there are four holes of 22 in diameter drilled into the web and the flange of the column.

A gusset is welded at the top part of the beam due to the FEM models. The decision was made to add a gusset for extra stiffness in the column.

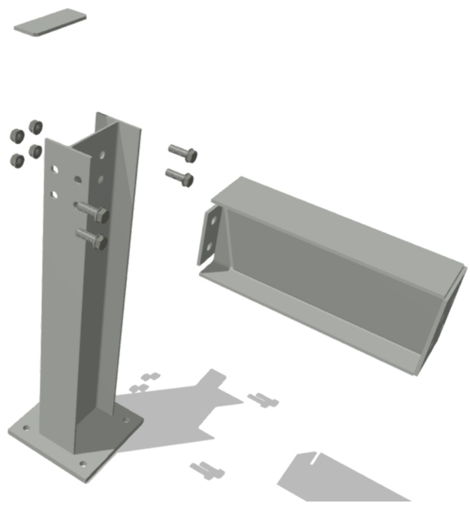


Figure 5 Exploded view of the Inventor model

The second I-beam has a length of 500mm and to both sides an endplate is welded. The one endplate is a full endplate welded to the flanges and the web of the I-beam, where the other endplate is a half endplate welded only to the web of the I-beam. In both endplates four holes of 22 in diameter is drilled at the same distance from each other as the holes in the column. The beam is bolted with four M20 bolts to the column.

The model will be tested in the two set-ups where the beam is mounted to the web and the column to research both ways of mounting the beam. Both mounts in used on the structures used by PBA on their structures. The same column with base in used to save material.

4.2.1 Masses

For FEA the mass of the model is important. Therefor the masses in Inventor are calculated and the fabricated model will be weight after assembly so that the masses can correspond to each other.

Part	Kg
Model with bolts	53,684
Model without bolts	52,392
Beam with endplates	15,431
Baseplate	11,113
Gusset	0,742
Half endplate	1,135
Full endplate	1,586
Beam	12,548
Column	24,934
Bolts	1,292

Table 1 Mass of the model according to Inventor 2015

4.3 FEA MODEL

Building the FEA model can be done in different ways, beam elements, plate elements and solid bricks. Due to the performance needed for solid modelling the FEA model will be done with beam elements and plate elements.

4.3.1 Beam elements

The FEA model with beam elements is a very simple model. The model is build up with two beam elements and an attachment between the two beams to simulate the endplate. On the node with the endplate a non-structural-mass is added in order to have the same mass as the other FEA models where the endplate can be modelled. The property of the beam can be defined as an I-beam 203x133x25. Depending on the plate element model the beam model is updated that the mass of both models are equal.

4.3.1.1 SUBDIVIDING

Setting up a FEA model with beam elements brings up a couple of choices. One of the choices is that the model can be divided into separate elements. In order to make the right choice a couple of dozen models were made to check what subdivision is accurate enough.
When subdividing a model the connection and the mass stay exactly the same. With the tool in Strand7 called "subdivide" the beams are divided in more beam elements.
The conclusion that can be made is that the model gives accurate enough measurements when the beams are divided into 7 elements. The precise results of the different models can be found in Appendix I: Subdividing Beam-elements results.

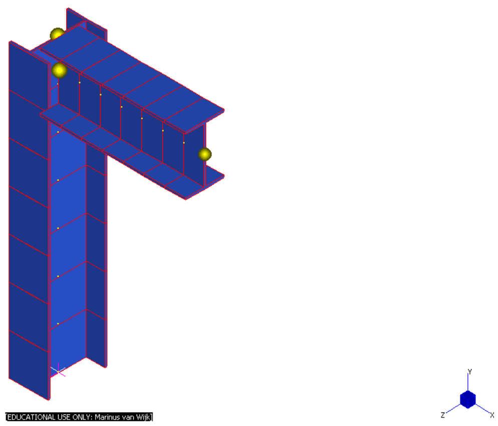


Figure 6 Beam FEA model with non-structural mass added at nodes and subdivided in 7 elements

4.3.1.2 FEA Results before measurements

The results of the beam model differ when mass is added to the nodes. The masses added to the nodes are the endplates, gusset and M20 bolts. The mass needs to match the mass of that of the plate-element model for the results to be reliable. The results are the final models after refining them in Strand7 but before measuring the fabricated model. The results of the four models are described below.

4.3.1.2.1 Full endplate – web mount (M1)

The results of the full endplate model have mass added to four nodes on the column. The connection used between the column and the beam is a beam-attachment with a direct moment connection. There are two modes that we are interested in and those are mode 2 and 7. These modes have a rotation around the z-axis and translation on the y-axis and can be found at 46.3 Hz and 187.7 Hz. Detailed results can be found in Appendix III: Results FEA Beam-elements.

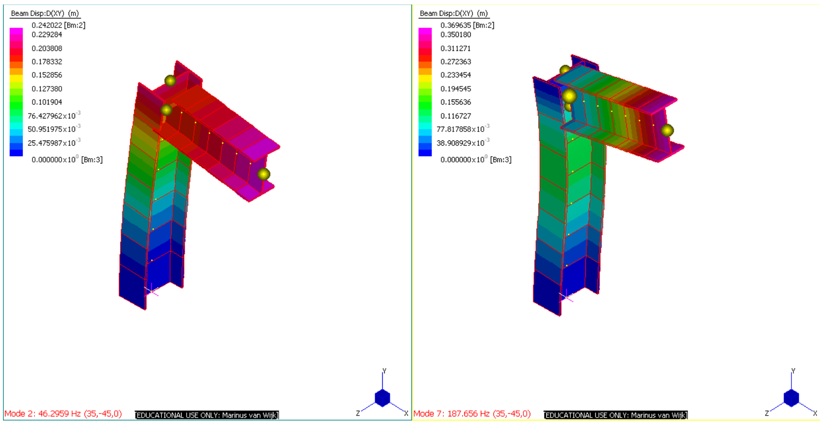


Figure 7 Full-endplate FEA model constructed with beam elements, displaying the two modes found.

4.3.1.2.2 Half endplate – web mount (M2)

The results of the half-endplate model are almost the same as the model with the full endplate. The difference is that the non-structural masses in the two nodes are the different to the full-endplate model and that a different connection is used. A flexible moment connection is used between the column and the beam. The connection-beam is determined with the plate model. The results found in the two modes are 45.8 Hz and 146.8 Hz. Detailed results can be found in Appendix III: Results FEA Beam-elements.

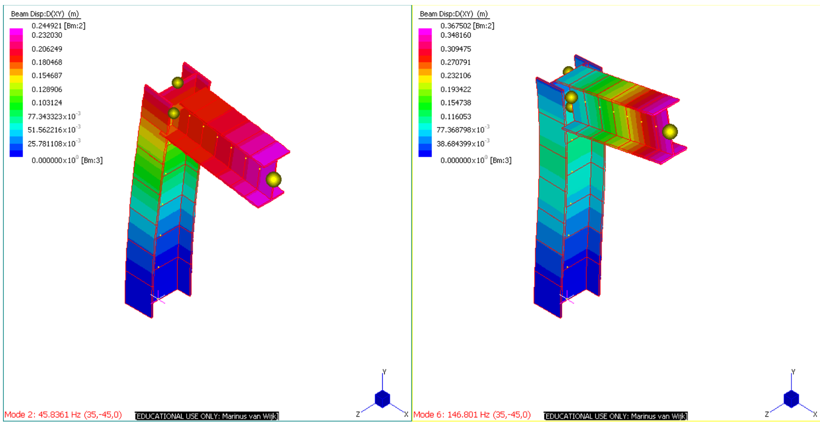


Figure 8 Half-endplate FEA model constructed with beam elements, displaying the two modes found.

4.3.1.2.3 FULL ENDPLATE – FLANGE MOUNT (M3)

The beam mounted with the full endplate to the flange of the column is a much stiffer system compared to the web mounted system. This means that natural frequencies found in this system will be much higher. The model doesn't need a gusset to simulate the stiffness of the I-beam in the plate-element model. This means that the only non-structural masses added are the endplates. The connection between the column and the beam is a beam-attachment with a direct moment connection. The results found in the two modes are 120.88 Hz and 276.75 Hz. Detailed results can be found in Appendix III: Results FEA Beam-elements.

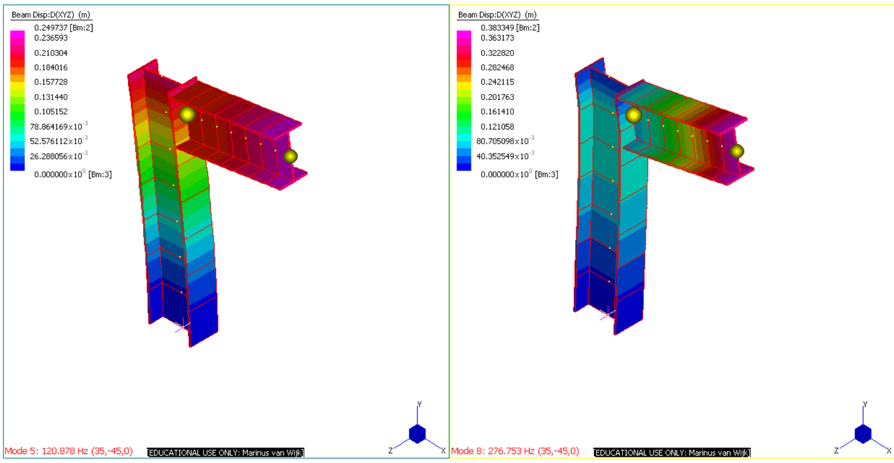


Figure 9 Full-endplate FEA model constructed with beam elements, displaying the two modes found.

4.3.1.2.4 HALF-ENDPLATE – FLANGE MOUNT (M4)

The half-endplate is a flexible connection and will result in different modes. The half-endplate model also doesn't need a gusset to simulate the correct stiffness of the I-beam therefor only the masses are swapped. The connection between the column and the beam is a beam-attachment with a flexible moment connection. The results found in the two modes are 109.02 Hz and 219.83 Hz. Detailed results can be found in Appendix III: Results FEA Beam-elements.

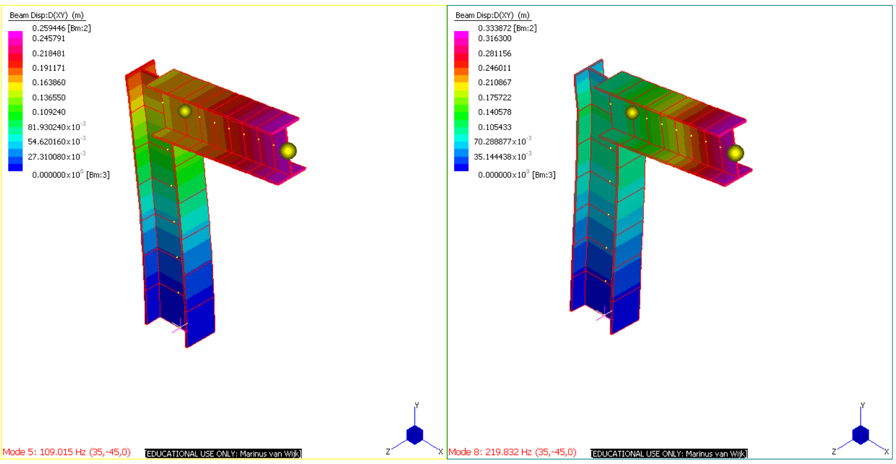


Figure 10 Half-endplate FEA model constructed with beam elements, displaying the two modes found.

4.3.2 Plate-elements

When modelling an I-beam in plate-elements the midsection of the plates of the beam is modelled in the FEA package.

4.3.2.1 AUTO MESHING VS MANUAL MESHING

The plate modelled FEA models can be modelled with two different methods. The first method is to model the beam manually in Strand7. After creating the main plates, the plates can be subdivided in smaller plates to increase accuracy. This method however is a lot of manual work when the model gets more complex with holes or different faces. The other is to create a surface 3D-model in Inventor and import the model as an IGN-file into Strand7. After the geometry is cleaned after import so that the free edges of the model are in the right position, the model can be meshed with the automesh feature of Strand7.

To determine if the two different methods correspond, a simple I-beam was modelled in both methods. The results of the natural frequencies calculated can be found in Table 2.

Mode	Automesh (Hz)	Manual mesh (Hz)	Difference (%)
1	86.15	86.03	-0,139
2	103.53	103.41	-0,116
3	212.19	212.21	0,009
4	244.15	244.57	0,172
5	325.20	325.78	0,178
6	450.20	451.64	0,319
7	464.01	464.56	0,118
8	501.83	502.10	0,054
9	542.66	541.19	-0,272
10	597.32	597.37	0,008

Table 2 Automesh modes vs. Manual mesh modes

Looking at the results we can conclude that auto meshing and manual meshing will give the same results. This means that for the future plate models they will all be modelled in Inventor and imported into Strand7. Detailed results can be found in Appendix II: Automesh vs Manual mesh results.

4.3.2.2 PLATE MODELS IN STRAND7

The plate-element models were modelled in Inventor and imported into Strand7. The mesh needs cleaning before the models can be auto meshed by Strand7. When the auto meshing is done the properties can be applied to the different plates.

The nodes of the holes that connect the beam to the column are connected with rigid links. The nodes at the end of the endplates are also connected to nodes of at the same height on the column. The reason for this is to simulate the connection when there is compression. Since the model is bolted connection, only tension will be applied to the bolts and not in the endplate connection to the column. The reason for applying the rigid joints at the endplate is to match the natural frequency of the beam-element model.

4.3.2.3 Results

The final results found of the four models are described below.

4.3.2.3.1 Full endplate – web mount (M5)

The web-mounted plate element model was defined to match the same results as the beam element. In order to do this, two gussets had to be placed at the top of the column to simulate the stiffness of the I-beam. The beam is attached with a rigid plate attachment to the column. The reason for this is that it saves time on relocating the mesh so that the nodes line up for a rigid link. The attachment adds calculation time but this is less than the time for aligning the mesh.

The results found with Strand7 after performing the natural frequency solver were modes 2 and 4 at 46.48 Hz and 186.35 Hz. Detailed results can be found in Appendix IV: Results FEA Plate-elements.

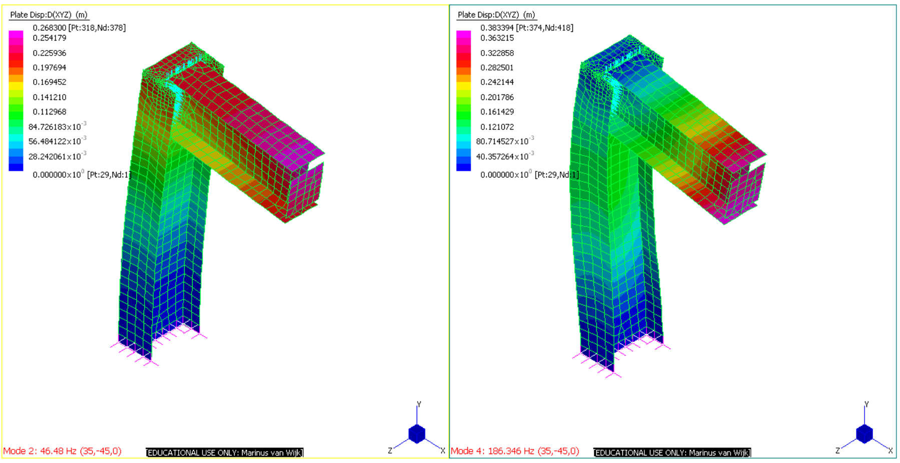


Figure 11 Full-endplate FEA model constructed with plate-elements, displaying the two modes found.

4.3.2.3.2 Half-endplate – web mount (M6)

The half-endplate gives total different results due to its flexibility. The beam-element model gave different results due to the connection used. When the connection was edited the results gave was in the same frequency range. The results found were mode 2 and 5 at 45.98Hz and 142.43Hz. Detailed results can be found in Appendix IV: Results FEA Plate-elements.

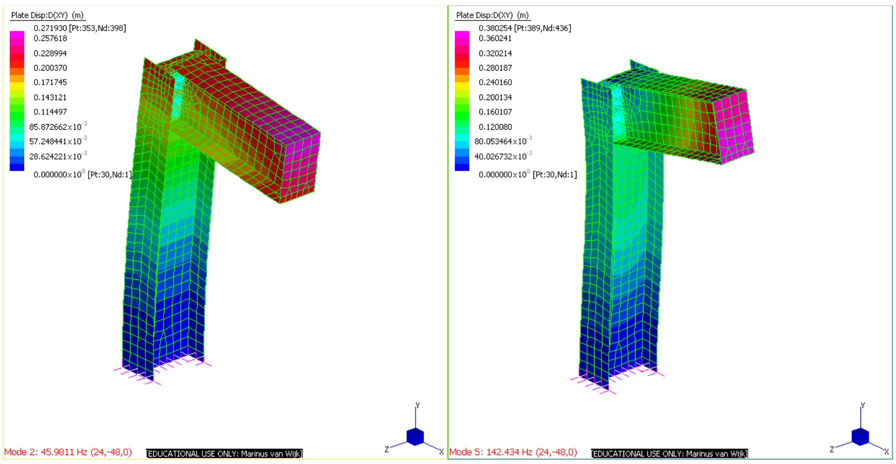


Figure 12 Half-endplate FEA model constructed with plate-elements, displaying the two modes found.

4.3.2.3.3 FULL-ENDPLATE – FLANGE MOUNT (M7)

The plate-element models of the flange-mounted beam were the easiest to match the results with the beam-elements models. The reason for this is that the beam is loaded on its strongest axis and didn't need any extra gussets to simulate the stiffness of the beam. Calculating the results find the natural frequency at mode 4 and 6 at 115.91 Hz and 270.40 Hz. Detailed results can be found in Appendix IV: Results FEA Plate-elements.

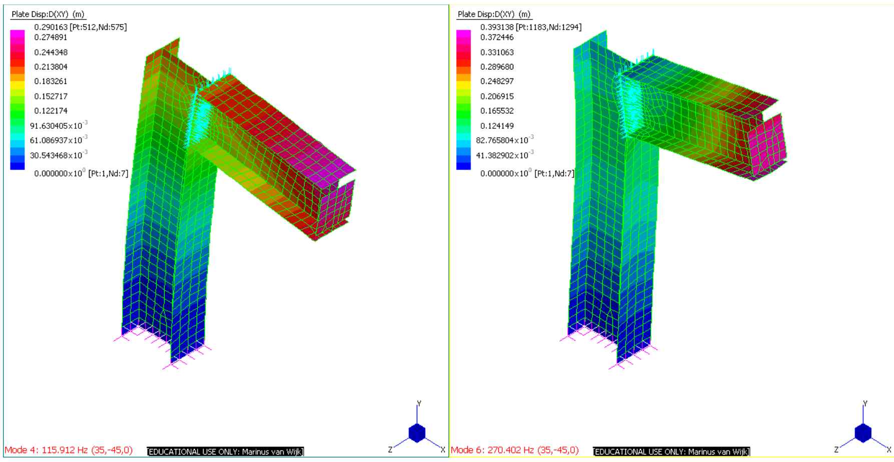


Figure 13 Full-endplate FEA model constructed with plate-elements, displaying the two modes found.

4.3.2.3.4 HALF-ENDPLATE – FLANGE MOUNT (M8)

For the half-endplate model a gusset was not needed on the column for the same reason as with the full-endplate. The results were found in mode 5 and 6 at 112.19 Hz and 213.88 Hz. Detailed results can be found in Appendix IV: Results FEA Plate-elements.

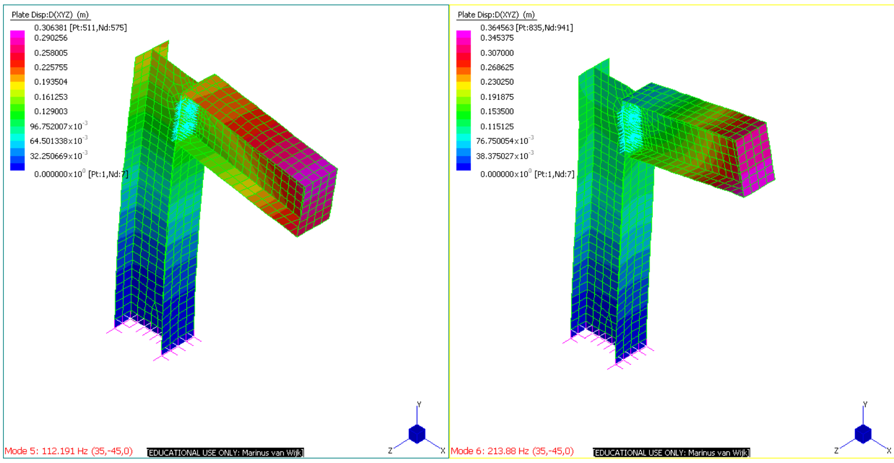


Figure 14 Half-endplate FEA model constructed with plate-elements, displaying the two modes found.

4.4 FEA RESULTS

The results below are found before the model was produced and measurements were made on the model. The FEA models will be checked if the results are similar to the measured natural frequencies on the fabricated model.

4.4.1 Web models

The full-endplate models (M5 and M1) gave roughly the same results. For this reason the beam-element model of half-endplate is edited and not the plate-element model.
Looking at the modes the models return the same mode shapes.
Setting up the models of the web-mounted models the beam-element model (M9) with the normal flexible moment connection gave total different results as the plate-element model (M6.) This can be explained due to the stiffness in the web of the column and the half-endplate that is much lower than the full-endplate. Adjusting the flexible moment connection to a much more flexible beam (model M2) to meet the frequency of the plate-element model (M6.) To make a good assumption, a measurement of the actual structure is needed.

Web - Final models							
Full endplate				*Half endplate*			
Rotation in the Z-axis, thus translation in the Y-axis of the models							
Model	**Mode**	**Hz**	**Disp-Y (m)**	**Model**	**Mode**	**Hz**	**Disp-Y (m)**
M5 Plate - G - Attach*	2	46,48	0,268	M6 Plate - G - Attach*	2	45,98	0,272
	4	186,35	0,383		5	142,43	0,380
M1 Beam - G - Attach**	2	46,296	0,242	M9 Beam - G - Attach***	2	45,93	0,241
	7	187,66	0,370		7	181,69	0,357
				M2 Beam - G - Attach****	2	45,84	0,245
					6	146,80	0,368

* Plate Attachment: Rigid with a moment connection
** Beam Attachment: Direct with a moment connection
*** Beam Attachment: Flexible with a moment connection
**** Beam Attachment: Flexible (beam 0.05x0.057)

Table 3 Final results on web-mounted models

4.4.2 Flange models

The two different flange-models, plate- and beam-element, returned results that matched. The mode shapes of the models also corresponded.

Flange - Final models							
Full endplate				*Half endplate*			
Rotation in the Z-axis, thus translation in the Y-axis of the models							
Model	**Mode**	**Hz**	**Disp-Y (m)**	**Model**	**Mode**	**Hz**	**Disp-Y (m)**
M7 Plate - NG - Attach*	4	115,91	0,290	M8 Plate - NG - Attach*	5	112,19	0,306
	6	270,40	0,393		6	213,88	0,365
M3 Beam - NG - Attach**	5	120,88	0,250	M4 Beam - NG - Attach***	5	109,02	0,259
	8	276,75	0,383		8	219,83	0,334

* Plate Attachment: Rigid with a moment connection
** Beam Attachment: Direct with a moment connection
*** Beam Attachment: Flexible with a moment connection

Table 4 Final results on flange-mounted models

5 Fabricating the Structure

The model was first modelled in Autodesk Inventor as described above. Detailed manufacturing drawings were made of the model so that the model can be fabricated at the production plant of PBA.
The manufacturing drawings can be found in Appendix V: CAD drawings.

5.1 The model

The first thing to manufacture is to cut the 203x133x25 I-beams in the correct length with a band saw. On the column of 1000mm is then marked where the holes of 22mm in diameter needs to be drilled.

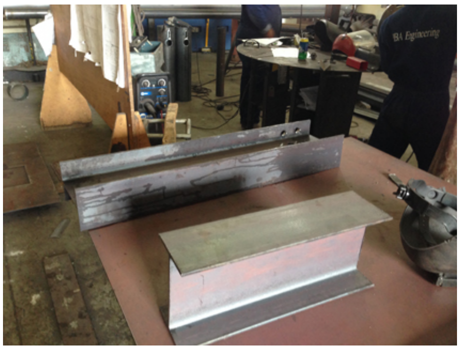


Figure 15 The two I-beams

The endplates and the baseplate are cut with a plasma cutter in the correct dimensions.
The endplates will then be welded to the beam but since the beam is to big to drill the 22mm holes in the endplates the holes are marked on the endplates and then drilled into the endplates. The baseplate is also marked where the 22mm holes in diameter needs to be drilled.

Figure 16 Drilling of the M20 clearance holes

After the drilling the parts are cleaned with a grinder and a pencil grinder so that the endplates can be welded to the beam. The column is welded to the middle of the baseplate and the gusset is also added to the column.

Figure 17 Cleaning of the parts

When the welding is done the model is cleaned with a grinder so that all sharp edges are burred. The model is now ready for assembly and uses four M20 bolts to connect the beam to the column.

Figure 18 Ready for welding

The intern did the manufacturing of the structure with guidance from factory workers at PBA Engineering. A professional welder at PBA Engineering however did the welding in order to make sure that poor welding doesn't affect the results.

5.2 Mass

After the fabrication the model is weighed in order to know the total mass of the column and beam. The mass is used to meet the mass of the FEA model.

The total mass of the beam and column is 54kg. This however is accurate to the kg since the scale used is precise to 1kg. Since this closely meets the mass that Inventor calculated, the mass Inventor calculated will be used for the FEA. The mass can be found in Table 1 on page 8.

5.3 Set up of the structure

The structure will be set up in four different models with the same column and beam as described earlier. In order to give an idea on how the structure will look in each set up a photo of each set up was made.

Figure 19 Web-mounted Full endplate

Figure 20 Web-mounted Half endplate

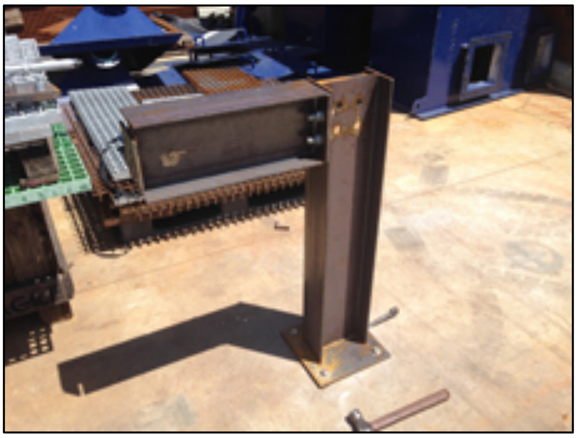


Figure 21 Flange-mounted Full endplate

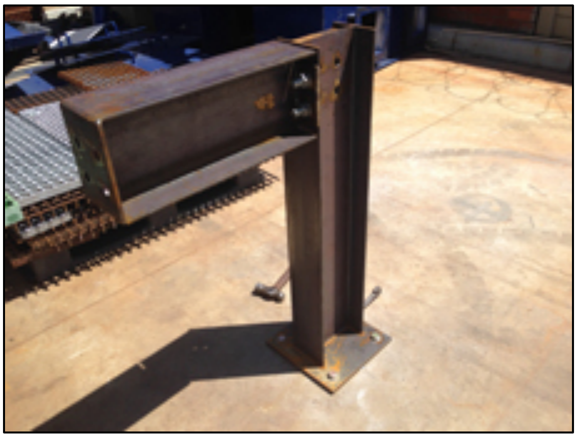


Figure 22 Flange-mounted Half endplate

6 Experimental Analysis

For the experimental analysis a procedure was set up first by doing a couple of measurements. After the procedure was set up the model was tested in different ways, which evolved over time due to the results found.

6.1 Measurement procedure

The first measurements that were made on the structure were to make sure the correct filters are used. Other measurements were to check the influence of a hammer by using a rubber hammer or an iron hammer. These measurements concluded in the following procedure.

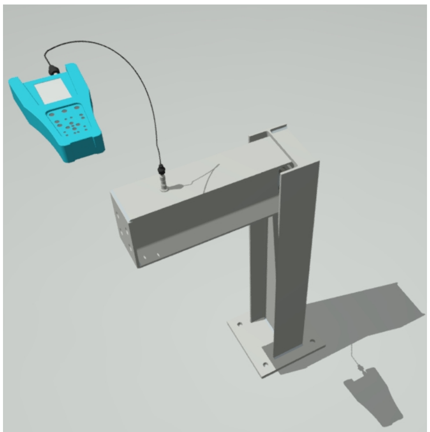


Figure 23 Set up for measuring vibrations

6.1.1 Conditions

The conditions of measuring the natural frequencies on the model are important so that they are consistent with the conditions used in Strand7.

- The torque on the bolts needs to be 125 Nm. (acc. De Beers spec 412Nm)
- The temperature needs to be between 15 and 30 degrees Celsius.
- The baseplate is fixed to a large mass.
- The rotation measured is the about the Z-axis of the model.
- The force applied will be on the outer end of the beam by a rubber hammer.

6.1.2 Measurement

Placement of the sensor will be at the outer edge of the beam. At this point the device can measure the deflection on the outer edge of the beam in the Y-direction.

6.1.2.1 *Filters*

The filters used for the VA-12 will be a FFT graph with:

- X axis; Frequency (Hz)
- Y axis; Acceleration (m/s/s)
- 800 Lines
- Frequency Span: 200Hz
- Operation type: LIN
- Window Function: Hanning
- Range will vary per measurement

6.1.3 Procedure

For accurate results the measurements will be done through the following procedure:

1. The baseplate with column is fixed to the ground, torqued at 125Nm.
2. The beam is mounted to the column in the desired position. The bolts are tightened with a torque of 125 Nm.
3. The sensor of the RION VA-12 is placed at the outer end of the beam.
4. Set time and date of the vibration meter and make sure the graph is plotted correctly. Calibrate the vibration meter. (Only the first time)
5. Start the measurement in the Y-direction by hitting the beam with a rubber hammer at its outer edge. Store the measurement data on the VA-12 and note the measured data in the excel sheet and the store-number of the VA-12 for later reference.

This procedure will be followed for all the measurements on the model.

6.2 FFT GRAPHS

The measurements on the model will be done with a RICO VA-12 vibration meter. This meter, as does many other meters, can place the frequency with the displacement in a Fast Fourier Transform (FFT) diagram. The Fourier transform family is used to represent periodic signals as the sum of properly chosen sinusoidal waves. The Fourier transform transforms the input of a function *N* in the time domain into two *N/2+1* point output signals in the frequency domain. The Fast Fourier Transform (FFT) is a complicated algorithm for a faster calculation of the discrete Fourier transforms (DFT.)

6.2.1 Reading FFT graphs

The graphs produced by the RION vibration meter is a FFT (Fast Fourier Transform) graph with on the horizontal axis the frequency (Hz) and on the vertical axis the displacement, velocity or acceleration.

The FFT is used to convert the data from time (s) to frequency (Hz) and makes it easy to read the natural frequencies of a model. The peaks in the graph where the displacement are at a maximum show where the natural frequencies of a model are.

6.3 MEASUREMENTS

The measurements were done according to the procedure described. The results are explained below. The FEA models results' that are plotted in the different charts was updated to match the mass of the structure.

Figure 24 The structure with the RION VA-12

6.3.1 Baseplate mounted to plate

The first set measurements on the structure were done while the structure was mounted to a large 20mm steel plate and weighed roughly 600kg. The choice for a metal plate was due to the lack of the possibility to anchor the structure to the ground at the factory at that time.

Figure 25 Structure mounted to 20mm plate

6.3.1.1 RESULTS

The results found are compared to the results found in FEA. The comparison can be found in the charts below. The difference for the web-mounted models was found roughly at 32%, where the flange-mounted models had a big difference of roughly 70%.

For the web-mounted with a full endplate the first mode was measured at 30.75 Hz, where the FEA found it at 45.83 Hz. This gives a difference of 33.5%. The second mode has an even larger difference of 45.8%. The results can be found in Chart 1.

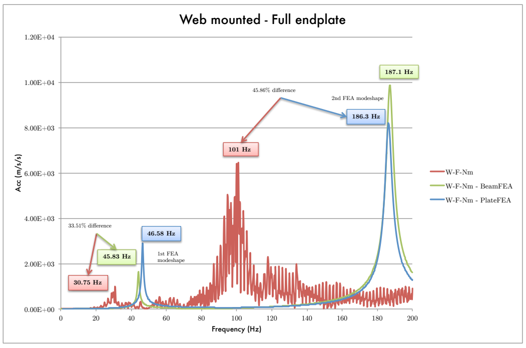


Chart 1 Web-mounted beam with a full-endplate. Mounted on a steel 20mm plate. (Appendix VI: Charts of measurements on plate)

The web-mounted with a half-endplate structure's first mode was measured at 31.25 Hz and the FEA found at 45.31 Hz. This results in a difference of 31.2%. The second mode has an even larger difference of 57.5%. The results can be found in Chart 2.

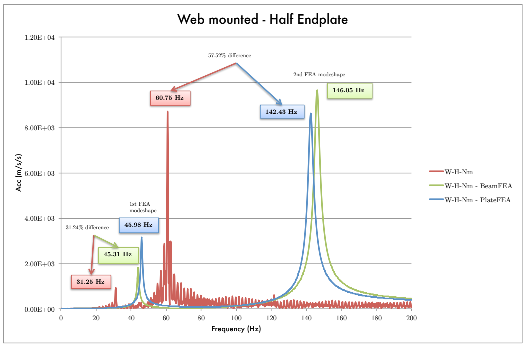


Chart 2 Web-mounted beam with a half-endplate. Mounted on a steel 20mm plate. (Appendix VI: Charts of measurements on plate)

The flange-mounted structures gave a total different result as expected. At full-endplate structure the first mode was measured at 34 Hz while the FEA result is found at 116.82 Hz. This resulted in a difference of 71.4%. The exact results can be found in Chart 3.

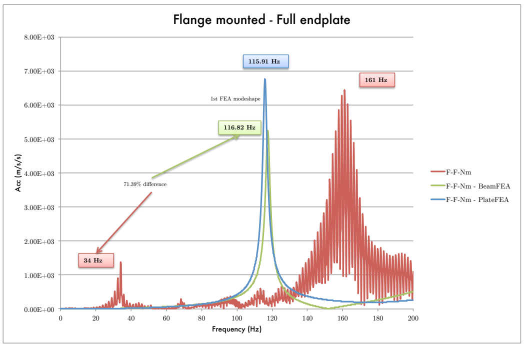


Chart 3 Flange-mounted beam with a full-endplate. Mounted on a steel 20mm plate. (Appendix VI: Charts of measurements on plate)

With the flange-mounted with a half-endplate connection almost the same results was captured in the measurement. The first mode was found at 33.25Hz and this gives a difference of 69.9% since the FEA result was found at 110.2Hz. The results can be found in Chart 4.

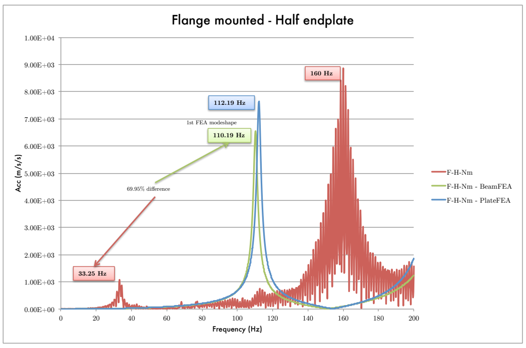


Chart 4 Flange-mounted beam with a half-endplate. Mounted on a steel 20mm plate. (Appendix VI: Charts of measurements on plate)

6.3.1.2 Conclusion

The frequencies found for the different structures were not the same as the natural frequencies calculated by FEA. The reason for the difference is mainly due to the plate the structure was mounted on. The plate was not anchored to the ground and therefore vibrated when the beam was hit. After modelling the plate in FEA in two test cases it became clear that the plate was largely influencing the results. In the models created in Strand7 two random points were used to simulate the points the plate was resting on. The results of the test cases can be found in Chart 5 & Chart 6. Concluding, for the next calculations the structure will be anchored in concrete.

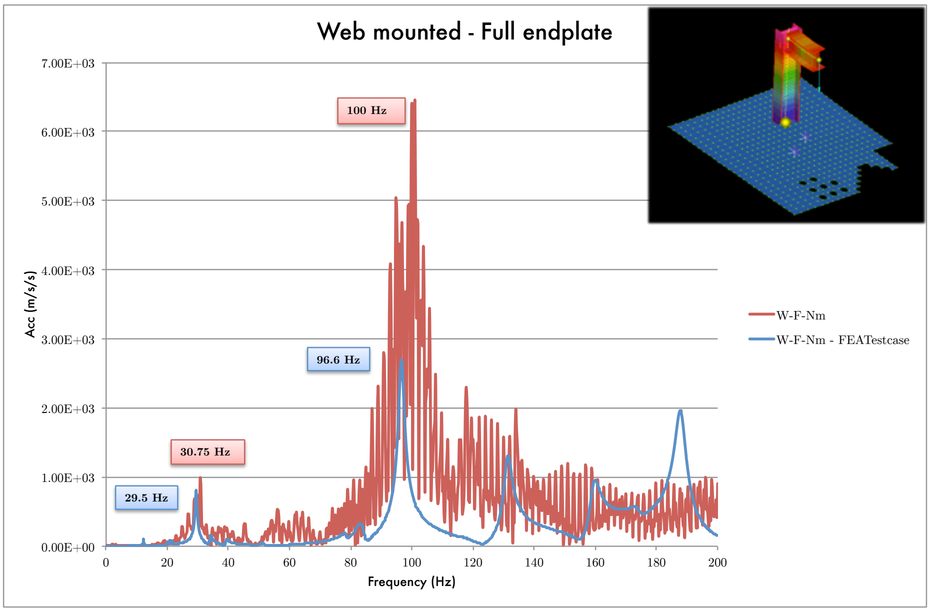


Chart 5 Web-mounted structure with full-endplate

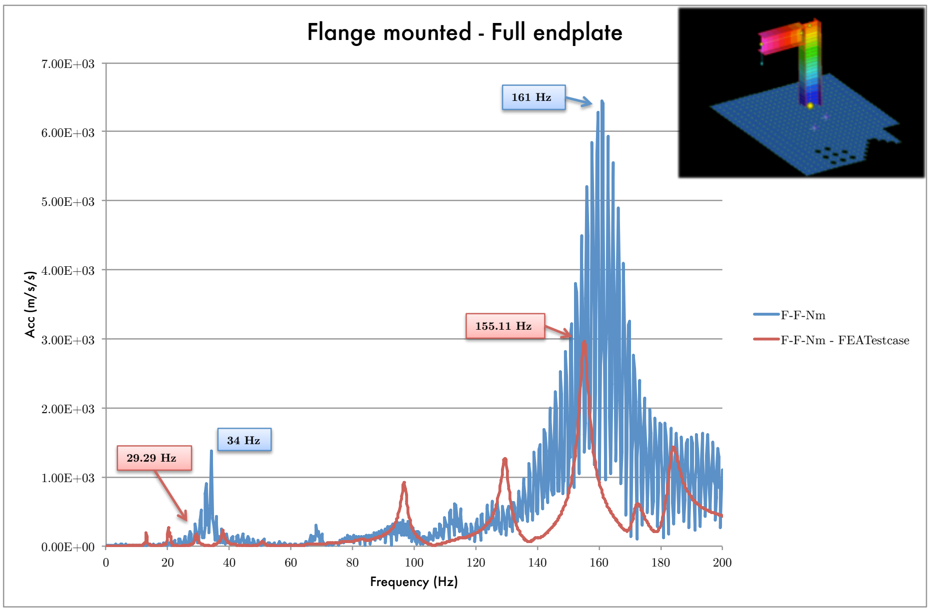


Chart 6 Flange-mounted structure with full-endplate

6.3.2 Baseplate anchored in concrete

The second set of measurements was done on the structure while it was anchored in concrete. Four M16 anchor bolts were used to bolt the baseplate to the concrete floor at the factory of PBA Engineering. The bolts were torqued at 125Nm in order to simulate a fixed connection as best as possible.

Figure 26 Structure anchored in concrete

6.3.2.1 Results

The overall measurement results improved compared to the first set of measurements. The results found were compared to the FEA data calculated in Strand7. The difference found between the results of the first mode for the web-mounted structures is roughly 16% and for the flange-mounted models 45%. This is a large improvement compared to the 30% on the web-mounted structures and the 70% on the flange-mounted structures.

For the web-mounted with a full endplate the first mode was measured at 39 Hz, where the FEA found it at 45.83 Hz. This gives a difference of 15.2%. The second mode has a difference of 38.8%. The results can be found in Chart 7.

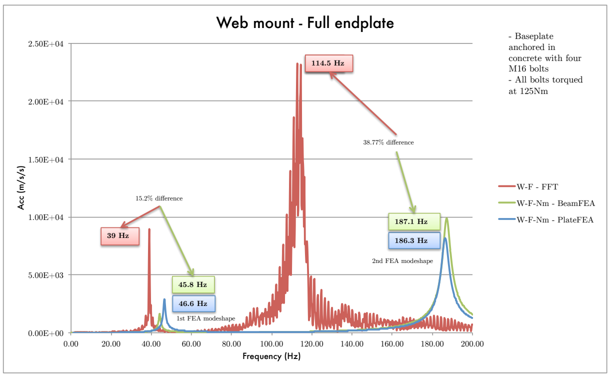


Chart 7 Web-mounted beam with a full-endplate. Anchored in concrete floor. (Appendix VII: Charts of measurements when anchored)

The web-mounted with a half-endplate structure's first mode was measured at 38 Hz and the FEA found at 45.31 Hz. This results in a difference of 17.4%. The second mode has a difference of 55.3%. The results can be found in Chart 8.

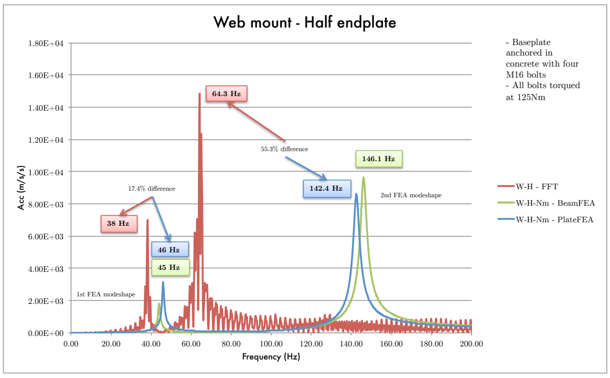


Chart 8 Web-mounted beam with a half-endplate. Anchored in concrete floor. (Appendix VII: Charts of measurements when anchored)

The flange-mounted structures result improved largely compared to the first set of measurements. At full-endplate structure the first mode was measured at 61 Hz while the FEA result is found at 116.82 Hz. This resulted in a difference of 46.8%. The exact results can be found in Chart 9.

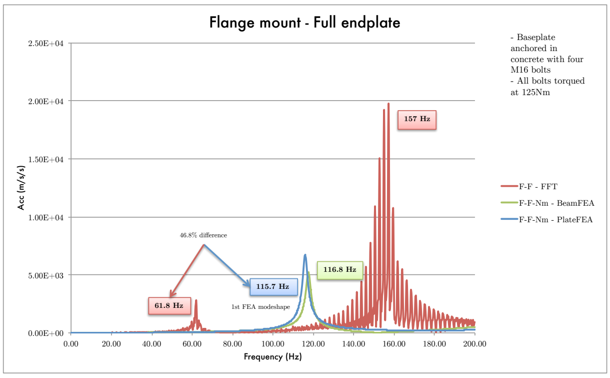


Chart 9 Flange-mounted beam with a full-endplate. Anchored in concrete floor. (Appendix VII: Charts of measurements when anchored)

With the flange-mounted with a half-endplate connection almost the same results was captured in the first measurement. The first mode was found at 61.3 Hz and this gives a difference of 44.3% since the FEA result was found at 110.2 Hz. The results can be found in Chart 10.

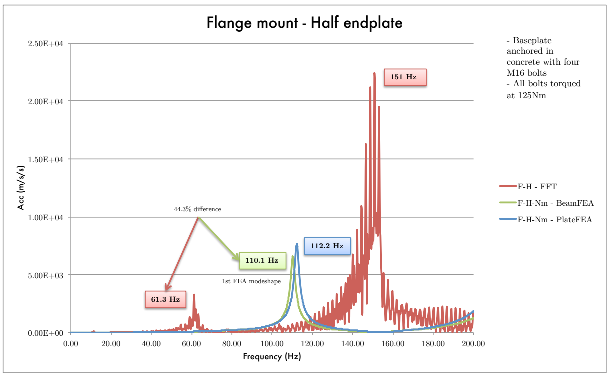


Chart 10 Flange-mounted beam with a half-endplate. Anchored in concrete floor. (Appendix VII: Charts of measurements when anchored)

6.3.2.2 Conclusion

The second set of measurements improved compared to the measurements taken on the 20mm plate. However the results are still not the same compared to the frequencies measured in FEA. More measurements need to be done to confirm the results found in FEA.

6.3.3 Baseplate anchored in concrete and extra wedges

Due to the welding the baseplate is not perfectly flat. This means that the structure will still vibrate at its base. In order to reduce the vibration from the baseplate to a minimum, wedges were slammed between the concrete and the baseplate. This was done after the anchor bolts were torqued and at locations where there was a gap between the baseplate and the concrete.

Figure 27 Structure anchored in concrete and extra wedges

6.3.3.1 RESULTS

The results in the third set of measurements didn't improve as much as expected. The first mode on the web-mounted structures gave the same results and for the flange-mounted structures minor improvement was achieved.

The web-mounted with a full endplate first mode was measured at 39 Hz, where the FEA found it at 45.83 Hz. This gives the same difference of 15.2% as measurement set two. The second mode has a difference of 41.2%. The results can be found in Chart 11.

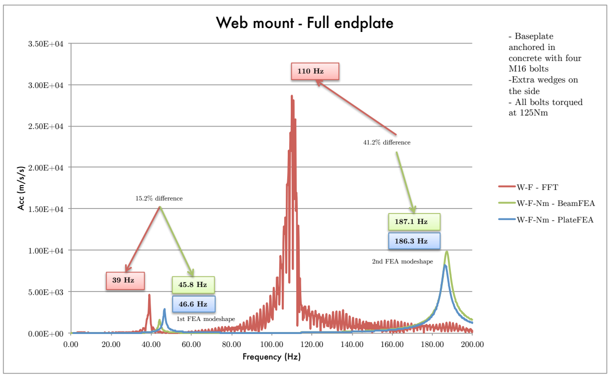


Chart 11 Web-mounted beam with a full-endplate. Anchored in concrete floor with wedges. (
Appendix VIII: Charts of measurements when anchored with extra wedges)

For the web-mounted with a half-endplate structure's first mode was measured at 38 Hz and the FEA found at 45.31 Hz. This results in the same difference of 17.4%. The second mode has a difference of 55.3%. The results can be found in Chart 12.

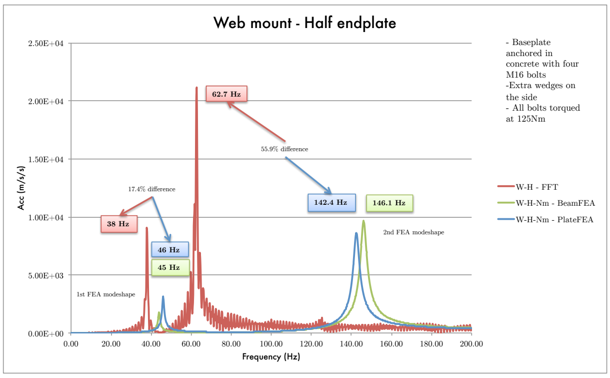


Chart 12 Web-mounted beam with a half-endplate. Anchored in concrete floor with wedges. (
Appendix VIII: Charts of measurements when anchored with extra wedges)

The flange-mounted structures result improved compared to the second set of measurements. At full-endplate structure the first mode was measured at 70.7 Hz while the FEA result is found at 116.82 Hz. This resulted in a difference of 38.9%. The exact results can be found in Chart 13.

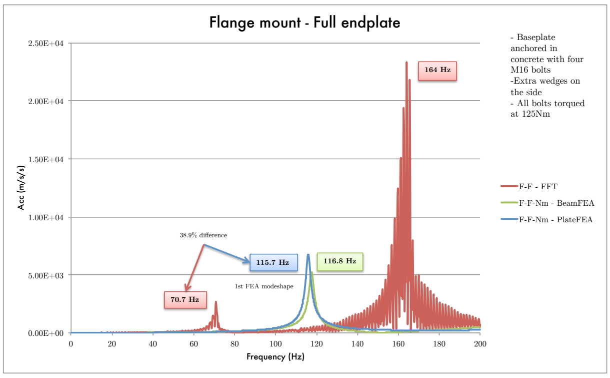


Chart 13 Flange-mounted beam with a full-endplate. Anchored in concrete floor with wedges. (*Appendix VIII: Charts of measurements when anchored with extra wedges)*

The flange-mounted with a half-endplate connection improved slightly compared to the second measurement. The first mode was found at 65.5 Hz and this gives a difference of 40.5% since the FEA result was found at 110.2Hz. The results can be found in Chart 14.

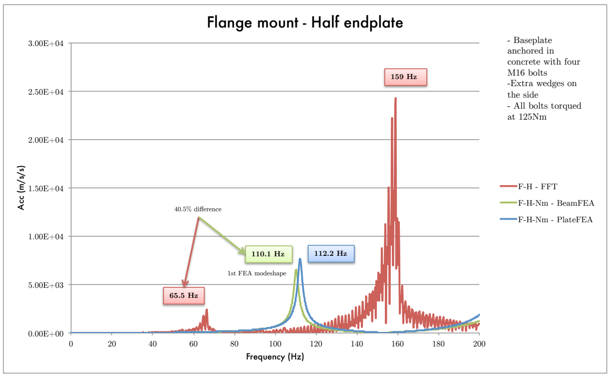


Chart 14 Flange-mounted beam with a half-endplate. Anchored in concrete floor with wedges. (*Appendix VIII: Charts of measurements when anchored with extra wedges)*

6.3.3.2 Conclusion

The results found in the third set of measurement still have a large difference when it is compared to the FEA results calculated in Strand7. The reason for this is that the FEA models are set up with a fixed restraint at the base of the column. The wedges added extra stability to the base but not enough to result in the same frequencies found in the FEA calculations.

Due to the results found, a FEA model was created to simulate the fixed connection that was used on the structure. At the position of the four anchor bolts in the baseplate four fixed nodes were placed in the FEA model. The nodes were fixed in the X, Y and Z-translation but not in the rotational degree. This resulted in the following charts.

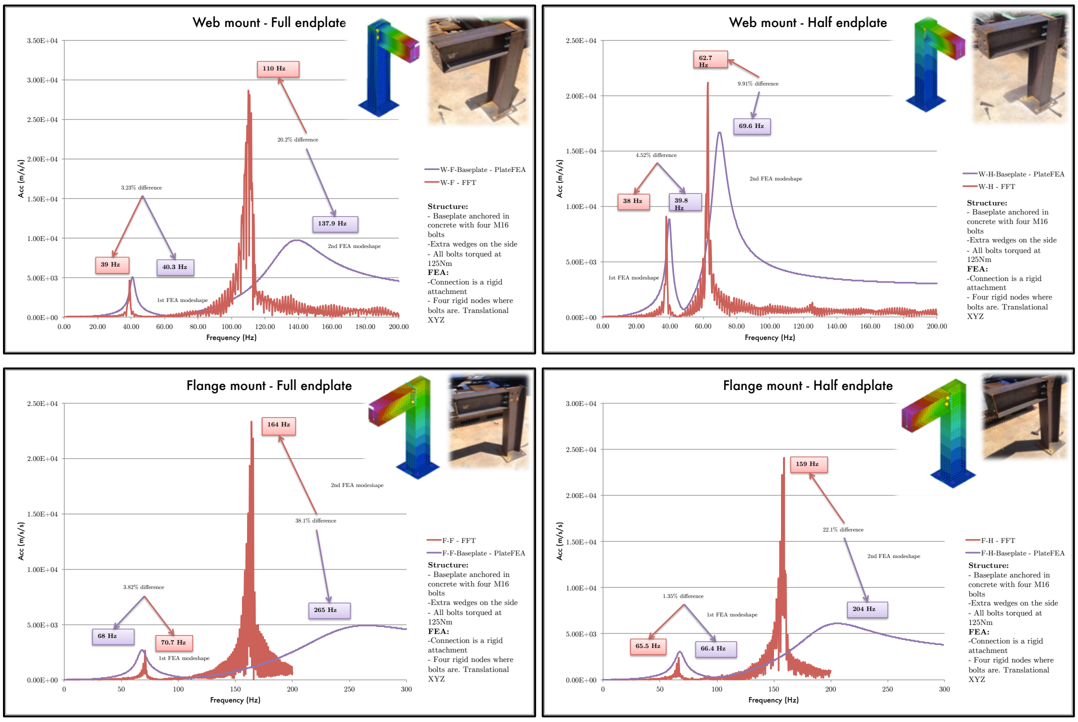


Chart 15 (a,b,c,d) FEA model updated to match the measurements (Appendix IX: Charts of measurements with improved FEA model)

The reason for the larger difference in the second mode has to do with the mode shape that is affected by the baseplate. The vibration of the baseplate will not be simulated correct because the plate is not solely fixed to the four nodes like the FEA model.

7 Results and discussion

The three measurement sets made it clear that the assumption of a fixed base is not correct when measuring vibrations on the structure. To get better results the baseplate was anchored on four more points close to the web of the column, making it a total of eight anchor bolts. This is to ensure that the effect of the baseplate is reduced to a minimum. The drawing of the baseplate was updated to add the four extra holes on the baseplate.

Figure 28 Eight bolts anchoring the baseplate

As shown in the picture the four anchor bolts added are located next to the web. The final measurement results are found the charts below.

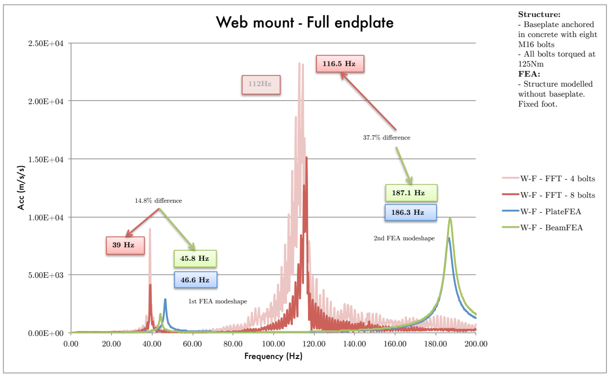


Chart 16 Web-mounted beam with a full-endplate. Eight times anchored in concrete floor. (Appendix X: Charts of measurements when eight times anchored in concrete)

The web-mounted with a full endplate (Chart 16) didn't improve with the extra anchor bolts. The difference still is roughly 14% in the first mode. The FEA plate-element model that was created before measuring the structure was modelled with two gussets to match the results found with the beam-element model. When leaving out the one gusset, as was done due to manufacturing reasons, the second natural frequency of the structure drops significantly. In the beam-element models the FEA package assumes a prefect beam and a perfect connection, not taking in account what will happen with the column if the beam is attached to the web. For this reason, when perfecting the plate-element models the results are closer to the reality. Installing a second gusset at the top of the column, near the bottom of the beam will improve the stiffness of the column and the forces will be transferred much more efficient.

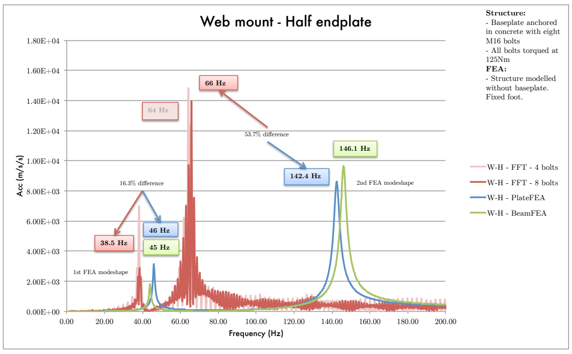


Chart 17 Web-mounted beam with a half-endplate. Eight times anchored in concrete floor. (Appendix X: Charts of measurements when eight times anchored in concrete)

The web-mounted with a half-endplate (Chart 17) results didn't improve either. The difference of the first mode is still roughly at 16%. This is due to the same reason given at the full-endplate model. The largely lower frequency, compared to the full-endplate, found at the second mode shape is the result of a smaller endplate that transfers the load onto the web of the column.

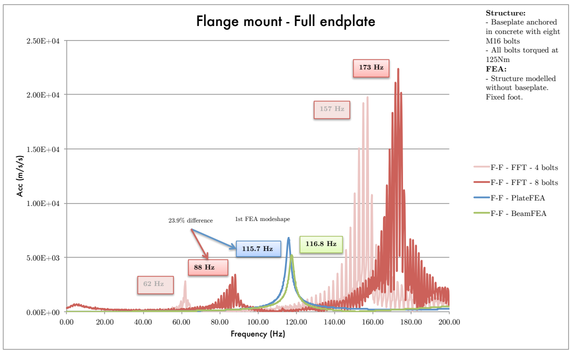


Chart 18 Flange-mounted beam with a full-endplate. Eight times anchored in concrete floor. (Appendix X: Charts of measurements when eight times anchored in concrete)

Compared to the normal anchored model the flange-mounted with full-endplate (Chart 18) improved significantly. The difference in now roughly 24% instead of 46.8% that was measured with the structure mounted with four anchor bolts. The improvement on the first mode is largely due to the stiffness added to the baseplate but isn't enough to make any assumptions on the connection. The reason firstly the baseplate that still vibrates and secondly the gusset that has an influence on the mode shapes.

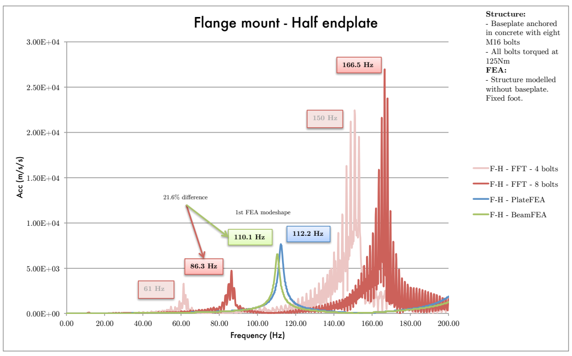


Chart 19 Flange-mounted beam with a Half-endplate. Eight times anchored in concrete floor. (Appendix X: Charts of measurements when eight times anchored in concrete)

The last structure, flange-mounted with a half-endplate (Chart 19) also improved largely to 21.6% for the same reason as with the full-endplate structure.

7.1 Discussion

Modelling the baseplate in FEA made it clear that the structure still isn't perfect. The structure with a beam still varies significantly in the second modes compared to the measurements.

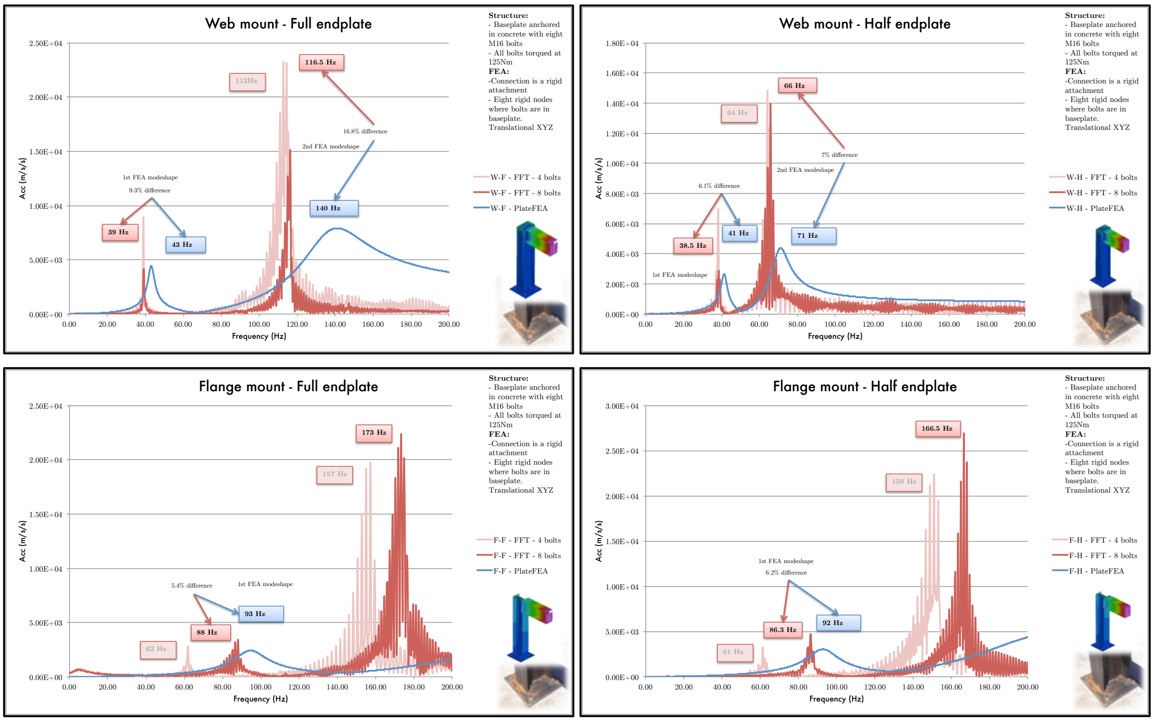


Figure 29 (a,b,c,d) FEA model updated including the baseplate. (Appendix XI: Charts of measurements when eight times anchored in concrete with improved FEA)

The reasons for the differences are mainly the way of anchoring the structure on all the structures. For the flange-mounted structures the gusset installed in the column is also influencing the measurements.

Since the results of the structures are not close enough to the results of first FEA models, PBA Projects should continue its research on the natural frequencies of the bolted connection. The best thing to do is to fabricate another column and do measurements on a supported beam. This, however, will bring extra costs since two beams are needed; one with on both ends a full-endplate and another with half-endplates. Also for the flange-mounted, two columns without gussets will need to be fabricated.

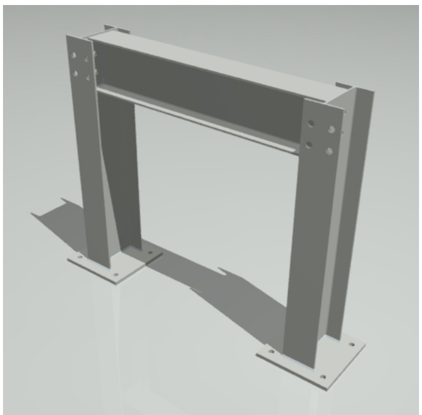


Figure 30 Supported beam with full-endplates

7.2 Planning and Project Management

The thesis mainly discusses the research done by the intern but in the background the intern managed the project. At the start of the graduation internship a planning was set up by the intern to give an approximate idea of the time needed to successfully finish the project.

During the first few weeks of the project the time needed to finish the first milestones was significantly less than expected in the Project brief. Understanding the FEA package PBA Projects uses, Strand7, proved to take less time than the two weeks that was planned for it. The research done at the start of the project was mainly to understand the background on FEM.

However, the time that was saved in the beginning of the project was needed for the measurements on the structure. Due to the complexity of the measurements more time was needed to compare the results to the FEA results.

The planning was adjusted during the project. While finalizing this thesis the project was at the exact point in the planning as expected at the beginning of the project. In Figure 31 the planning can be found where the dark green is the progress of the project, light green the initial planning and blue needs to happen.

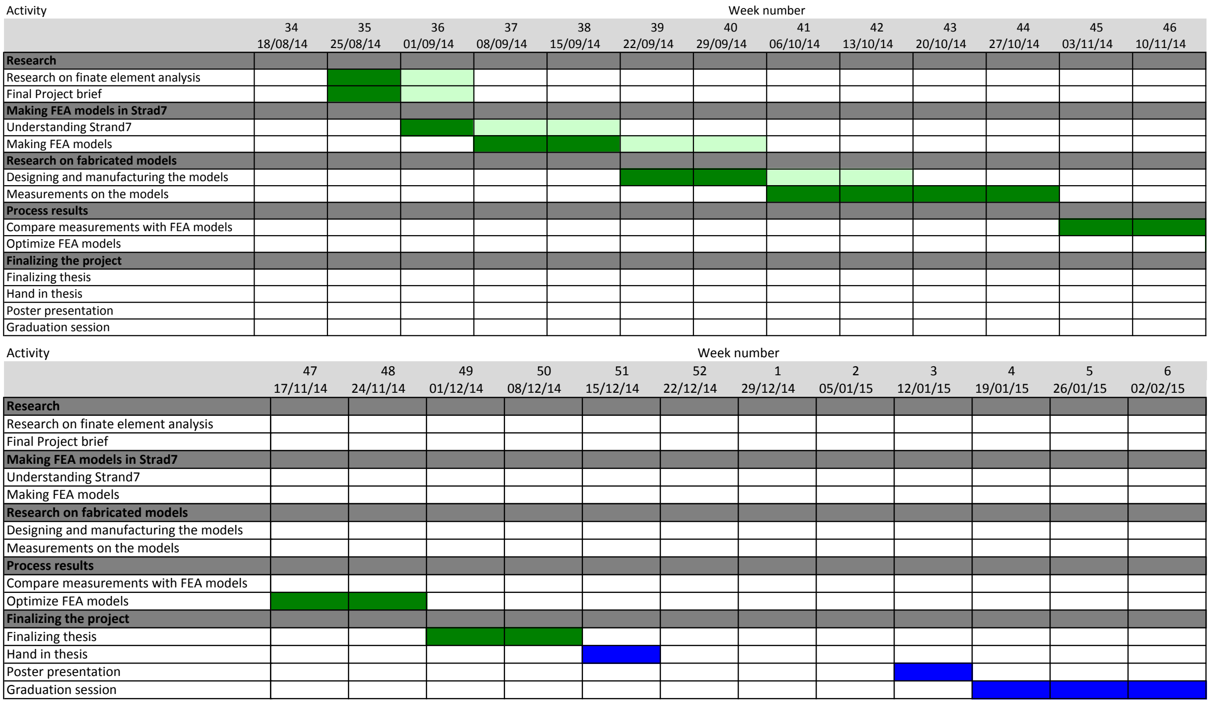


Figure 31 Planning of the project

8 Conclusion

During this thesis an in-depth research was done on the natural frequency on a simple structure. The following can be concluded:

- Assumptions on fixed nodes have a huge influence on the measurements of the structures.
- The endplate gives a notable difference in the natural frequency found in the beams, especially the higher modes.
- Frequencies found with web-mounted conclude that the plate-element model calculates the frequency nearest to the measured natural frequency. Therefor the frequencies found with plate-element models can be used to calibrate the beam-element models.
- The structure is stiffer than FEA calculates.

The natural frequencies found on the structure compared to the FEA models are the following:

	Full-endplate (FEA)		*Full-endplate (Measured)*		*Difference (%)*		*Half-endplate (FEA)*		*Half-endplate (Measured)*		*Difference (%)*	
Web-mounted	43	140	39	116	*9.3*	*16.8*	41	71	38.5	66	*6.1*	*7*
Flange-mounted	93	266	88	173	*5.4*	*34.9*	92	207	86.3	166.5	*6.2*	*19.6*

Table 5 First and second natural frequencies found in Hz

For further research a simply supported beam needs to be constructed. This structure will be useful to determine the use of end-releases used on beam structures in FEA. The beam should not be too long to keep the effect of the vibrations of the baseplates on the beam to minimum. This structure will need to be anchored the same way the single column was fixed to a concrete slab.

Bibliography

V. Adams, A. Askenazi, *Building Better Products with Finite Element Analysis.* Santa OnWord Press, 1999.

AA BPG S02, *Guideline for the vibration design of structures.* AngloAmerican, 2014.

Internet

https://www.asme.org	Information on FEA
http://www.pbaprojects.co.za	PBA Projects
http://www.pbaengineering.co.za	PBA Engineering
http://www.strand7.com	Help on Strand7, Tutorials
http://www.journals.elsevier.com/finite-elements-in-analysis-and-design	Papers on Finite Element Analysis
http://www.colorado.edu/MCEN/MCEN4173/lecture_notes.html	Information on FEA

Competences

The Hague University wants every student to give an indication of the level of the internship and the roles that assigned to the student. Che Smit of PBA Projects approved the table to acknowledge that the indicated roles were fulfilled.

The student managed the whole research project. Setting up a planning, project brief and maintaining the planning was a challenge at the end of the project. Due to the complexity of the research the measurements were not simple to read out, therefore drawing conclusions was not easily done. After a couple of measurements more research had to be done.

The student has developed new skills in managing a research. Secondly he developed new skills in Finite Element Analysis, learning general modelling rules and researching new areas in FEA.

Roles		***Researcher***	***Designer***	***Advisor***	***Management***	***Project manager***	***Entrepreneur***
Competences for Mechanical Engineering and general competences							
Nr.	**Competences for Mechanical Engineering**						
1	Project management (organize, plan, implement, report)					3	
2	Perform research	4					
3	Set up a product definition, action plan and a set of requirements for sustainable product or process					2	
4	Realize a functional and sustainable product or process	3					
5	Realize a detailed design of a sustainable product or process						
6	Realize a prototype or model of a sustainable product or process	3				2	
7	Prepare a process						
8	Manufacturing of a sustainable product						
9	Handle or maintain a product or process						
Nr.	**General HBO competences**						
10	Critically evaluate (analytical and problem solving skills, substantiate the choices and judgement)	4				3	
11	Tackles a problem systematically (Creative and project-oriented attitude)	4				3	
12	Teamwork (People skills)	3				3	
13	Personal and professional development	3				3	
14	Work responsible	3				3	
15	Functions in an international or multicultural environment	3				3	

Appendix

The following appendices are attached:

1. Appendix I: Subdividing Beam-elements results
2. Appendix II: Automesh vs Manual mesh results
3. Appendix III: Results FEA Beam-elements
4. Appendix IV: Results FEA: Plate-elements
5. Appendix V: CAD drawings
6. Appendix VI: Charts of measurements on plate
7. Appendix VII: Charts of measurements when anchored
8. Appendix VIII: Charts of measurements when anchored with extra wedges
9. Appendix IX: Charts of measurements with improved FEA model
10. Appendix X: Chart of measurements when eight times anchored in concrete
11. Appendix XI: Chart of measurements when eight times anchored in concrete with improved FEA

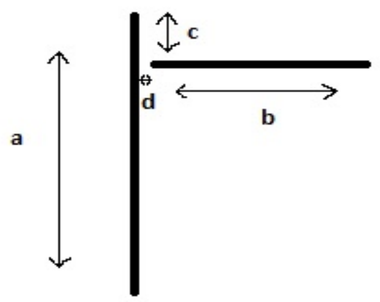
APPENDIX I: SUBDIVIDING BEAM-ELEMENTS RESULTS

Web - Beam Elements models							
Full endplate				*Half endplate*			
Rotation in the Z-axis, thus translation in the Y-axis of the models							
Subdivided	**Mode**	**Hz**	**%-rot**	**Subdivided**	**Mode**	**Hz**	**%-rot**
1	2	45,99	80,00	1	2	45,55	97,63
1	5	241,52	0,64	1	6	169,78	2,35
1	7	1029,7	0,03	1	7	895,84	0,01
1	9	1706,10	0,64	1	9	1079,26	0,00
2	2	45,97	73,56	2	2	45,53	86,95
2	6	212,43	1,56	2	6	159,83	3,01
3	2	45,96	81,06	3	2	45,51	91,54
3	7	201,48	0,47	3	6	154,45	1,86
4	2	45,95	82,81	4	2	45,50	91,98
4	7	193,90	0,63	4	6	150,43	1,92
5	2	45,95	83,78	5	2	45,50	92,24
5	7	188,97	0,89	5	6	147,71	2,14
6	2	45,96	84,47	6	2	45,51	92,55
6	7	186,99	1,12	6	6	146,58	2,33
7	2	45,97	84,96	7	2	45,52	92,92
7	7	188,14	1,31	7	6	147,14	2,46
13	2	45,90	88,49	13	2	45,43	94,08
13	7	176,99	1,82	13	5	141,09	2,94
50	2	45,96	90,80	50	2	45,51	95,44
50	7	187,02	2,16	50	7	146,47	3,10
Plate	2	46,48	95,85	Plate	2	45,98	95,37
Plate	4	186,35	2,83	Plate	5	142,43	3,48

Global information

Temperature	293,00 K	
Gravity (-Y)	9,81 m/s²	
Mass Plate element		**Mass Beam element**
Full Endplate	1,548 kg	1,548 kg
Half Endplate	1,174 kg	1,174 kg
Total Endplates	2,722 kg	2,722 kg
Beam	12,559 kg	12,639 kg
Column	25,030 kg	25,277 kg
Gusset 2x	0,819 kg (x2)	0,819 kg (x2)
I-Beam total	37,589 kg	37,916 kg
Total	41,949 kg	42,276 kg

Dimensions



Dimension

a	1,0000	m
b	0,5000	m
c	0,1016	m
d	0,0040	m

Dimensions are used for the placement of the nodes

Conclusion: Subdividing the beam-elements into 7 elemenst is sufficient for the accuracy. Due to the connection used at the half element the subdivision must be done very precisely so that the connection is inbetween two nodes.

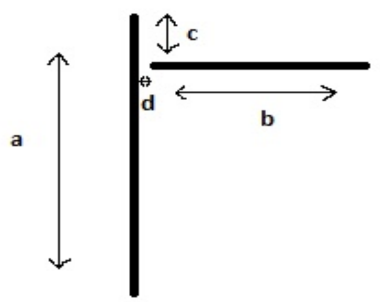
Flange - Beam Elements models

Full endplate				*Half endplate*			
Rotation in the Z-axis, thus translation in the Y-axis of the models							
Subdivided	**Mode**	**Hz**	**%-rot**	**Subdivided**	**Mode**	**Hz**	**%-rot**
1	4	121,27	79,86	1	4	110,43	95,29
1	5	431,45	0,06	1	6	339,11	1,77
1	8	1142,90	0,00	1	9	1055,90	0,00
1	9	2037,20	1,04	1	10	1810,70	0,01
2	5	120,74	73,92	2	5	109,62	85,60
2	7	383,74	0,18	2	7	308,57	3,16
3	5	120,42	81,63	3	5	108,92	90,11
3	8	340,40	0,01	3	8	273,49	2,86
4	5	120,23	83,48	4	5	108,34	89,93
4	9	304,29	0,07	4	8	243,69	3,91
5	5	120,22	84,47	5	5	108,05	89,48
5	8	280,61	0,17	5	8	224,27	4,98
6	5	120,44	85,20	6	5	108,25	89,61
6	8	271,00	0,22	6	8	216,08	5,38
7	5	120,88	85,76	7	5	109,01	90,71
7	8	276,75	0,16	7	8	219,83	4,77
13	5	118,43	88,26	13	5	104,68	85,72
13	8	232,71	1,62	13	7	188,30	11,46
50	5	120,64	91,77	50	5	108,84	93,16
50	9	282,36	0,56	50	8	224,45	5,52
Plate	4	115,91	30,77	Plate	5	112,19	27,27
Plate	6	270,40	23,17	Plate	6	213,88	28,23

Global information

Temperature	293,00 K	
Gravity (-Y)	9,81 m/s^2	
Mass Plate element		**Mass Beam element**
Full Endplate	1,548 kg	1,548 kg
Half Endplate	1,174 kg	1,174 kg
Total Endplates	2,722 kg	2,722 kg
Beam	12,559 kg	12,639 kg
Column	25,030 kg	25,277 kg
Gusset	0,000 kg	0,000 kg
I-Beam total	37,589 kg	37,916 kg
Total	40,311 kg	40,637 kg

Dimensions



Dimension

a	1,0000	m
b	0,5000	m
c	0,1016	m
d	0,1056	m

Dimensions are used for the placement of the nodes

Conclusion: Subdividing the beam-elements into 7 elemenst is sufficient for the accuracy. Due to the connection used at the half element the subdivision must be done very precisely so that the connection is inbetween two nodes.

APPENDIX II: AUTOMESH VS MANUAL MESH RESULTS

RESULTS AUTOMESH I BEAM

```
____________________________________________
*Solution commenced on 15/09/2014 at 10:39:46

 Strand7 [2.4.6][Solver Build: 24131212] (32-Bit) [EDUCATIONAL USE ONLY]

 ANALYSIS TYPE                : NATURAL FREQUENCY

 COMPUTER NAME                : pbapdesk001
 USER LOGON NAME              : marinusvanwijk
 CPU                          : Intel(R) Pentium(R) 4 CPU 2.40GHz
 USABLE PHYSICAL MEMORY       : 4.0 GB
 USABLE VIRTUAL MEMORY        : 3.0 GB

 MODEL FILE                   : "Y:\Desktop\Strand7\ModelPBA\Thesis\Automesh I Beam.st7"
 RESULT FILE                  : "Y:\Desktop\Strand7\ModelPBA\Thesis\Automesh I Beam.NFA"
 SCRATCH PATH                 : "C:\users\crossover\Strand7\Tmp\"

 TOTALS
 Nodes                        : 544
 Beams                        :   0
 Plates                       : 495
 Bricks                       :   0
 Links                        :   0

 SOLVER UNITS
 Length                       : m
 Mass                         : kg
 Force                        : N
 Stress                       : Pa

 FREEDOM CASE                 : "Freedom Case 1"

 MASS MATRIX OPTION
 Plate Elements               : Consistent
 Global Matrix                : Full
 Non-Structural Mass          : Added (from the following cases)
                              : "Load Case 1"

 STORAGE SCHEME               : Sparse
 SORTING METHOD               : AMD

 NUMBER OF EQUATIONS          : 3168
 MATRIX FILL-IN               : 70.9%
 [K] MATRIX SIZE              : 2.1 MB
 [M] MATRIX SIZE              : 639.8 KB
 OPTIMUM RAM NEEDED           : 960.0 KB
 FREE SCRATCH SPACE           : 247.4 GB

 Reducing 3168 Equations (Using 2.2 MB RAM)...

 MAXIMUM PIVOT                : 3.866269E+09 (Node 49 DZ)
 MINIMUM PIVOT                : 3.273766E+03 (Node 4 RY)
```

```
 NODAL DISPLACEMENT COMPONENTS USED IN STARTING VECTOR
     211[DX]      230[DX]      236[DX]      205[DX]      141[DX]      235[DX]
     159[DX]      160[DX]      229[DX]      214[DX]      220[DX]      212[DX]
     257[DX]      178[DX]      194[DX]

 SUBSPACE ITERATION
 Natural Frequencies          : 10
 Mass Degrees of Freedom      : 2916
 Subspace Dimension           : 16
 Convergence Tolerance        : 1.000000E-05
 Frequency Shift              : 0.000000E+00 Hz

THE FIRST 10 EIGENVALUES HAVE CONVERGED

 FINAL FREQUENCY RESULTS
 Mode        Eigenvalue       Frequency (rad/s)    Frequency (Hz)
    1      2.93013591E+05      5.41307298E+02      8.61517322E+01
    2      4.23193691E+05      6.50533390E+02      1.03535605E+02
    3      1.77749559E+06      1.33322751E+03      2.12189748E+02
    4      2.35325630E+06      1.53403269E+03      2.44148886E+02
    5      4.17496302E+06      2.04327263E+03      3.25196938E+02
    6      8.00165304E+06      2.82871933E+03      4.50204664E+02
    7      8.49990256E+06      2.91545924E+03      4.64009749E+02
    8      9.94182569E+06      3.15306608E+03      5.01826052E+02
    9      1.16256822E+07      3.40964547E+03      5.42661931E+02
   10      1.40855769E+07      3.75307566E+03      5.97320543E+02

 Mass Participation Factors and Effective Modal Damping...

 TOTAL MASS (MX,MY,MZ)        : ( 2.461123E+01, 2.461123E+01, 2.461123E+01)

 MODE PARTICIPATION FOR TRANSLATIONAL EXCITATION
 Mode     Frequency     Modal Mass     Modal Stiff     PF-X       PF-Y       PF-Z
            (Hz)          (Eng)          (Eng)          (%)        (%)        (%)
    1    8.6152E+01    5.7815E+00    1.6941E+06     62.720      0.000      0.000
    2    1.0354E+02    5.6019E+00    2.3707E+06      0.000      0.000      0.000
    3    2.1219E+02    6.7996E+00    1.2086E+07      0.000     65.579      0.000
    4    2.4415E+02    3.7568E+00    8.8407E+06      0.053      0.000      0.000
    5    3.2520E+02    3.6287E+00    1.5150E+07      5.499      0.000      0.000
    6    4.5020E+02    4.0416E+00    3.2339E+07      0.000      0.000      0.000
    7    4.6401E+02    2.8326E+00    2.4076E+07      0.279      0.000      0.000
    8    5.0183E+02    2.1425E+00    2.1301E+07      0.000      0.000      0.000
    9    5.4266E+02    6.4541E+00    7.5034E+07     15.213      0.000      0.000
   10    5.9732E+02    6.8688E+00    9.6751E+07      0.000      0.000      0.000
 ----------------------------------------------------------------------------
 TOTAL TRANSLATIONAL MASS PARTICIPATION FACTORS    83.764     65.579      0.000

 MODE PARTICIPATION FOR ROTATIONAL EXCITATION
 Mode     Frequency     Modal Mass     Modal Stiff     PF-RX      PF-RY      PF-RZ
            (Hz)          (Eng)          (Eng)          (%)        (%)        (%)
    1    8.6152E+01    5.7815E+00    1.6941E+06      0.000     14.620      0.000
    2    1.0354E+02    5.6019E+00    2.3707E+06      0.000      0.000     63.679
    3    2.1219E+02    6.7996E+00    1.2086E+07     15.496      0.000      0.000
    4    2.4415E+02    3.7568E+00    8.8407E+06      0.000      0.442      0.000
    5    3.2520E+02    3.6287E+00    1.5150E+07      0.000     10.829      0.000
    6    4.5020E+02    4.0416E+00    3.2339E+07      0.000      0.000      1.109
    7    4.6401E+02    2.8326E+00    2.4076E+07      0.000      0.317      0.000
    8    5.0183E+02    2.1425E+00    2.1301E+07      0.000      0.000      7.154
```

```
   9    5.4266E+02    6.4541E+00    7.5034E+07     0.000    30.760     0.000
  10    5.9732E+02    6.8688E+00    9.6751E+07     0.000     0.000    10.539
 ------------------------------------------------------------------------------
 TOTAL ROTATIONAL MASS PARTICIPATION FACTORS       15.496    56.969    82.481

 TOTAL CPU TIME              : 2.010 Seconds ( 0:00:02)
```

RESULTS MANUAL MESH I BEAM

```
____________________________________________
*Solution commenced on 15/09/2014 at 10:42:00

 Strand7 [2.4.6][Solver Build: 24131212] (32-Bit) [EDUCATIONAL USE ONLY]

 ANALYSIS TYPE               : NATURAL FREQUENCY

 COMPUTER NAME               : pbapdesk001
 USER LOGON NAME             : marinusvanwijk
 CPU                         : Intel(R) Pentium(R) 4 CPU 2.40GHz
 USABLE PHYSICAL MEMORY      : 4.0 GB
 USABLE VIRTUAL MEMORY       : 3.0 GB

 MODEL FILE                  : "Y:\Desktop\Strand7\ModelPBA\Thesis\Manual Mesh I beam.st7"
 RESULT FILE                 : "Y:\Desktop\Strand7\ModelPBA\Thesis\Manual Mesh I beam.NFA"
 SCRATCH PATH                : "C:\users\crossover\Strand7\Tmp\"

 TOTALS
 Nodes                       : 779
 Beams                       :   0
 Plates                      : 720
 Bricks                      :   0
 Links                       :   0

 SOLVER UNITS
 Length                      : m
 Mass                        : kg
 Force                       : N
 Stress                      : Pa

 FREEDOM CASE                : "Freedom Case 1"

 MASS MATRIX OPTION
 Plate Elements              : Consistent
 Global Matrix               : Full
 Non-Structural Mass         : Added (from the following cases)
                             : "Load Case 1"

 STORAGE SCHEME              : Sparse
 SORTING METHOD              : AMD

 NUMBER OF EQUATIONS         : 4560
 MATRIX FILL-IN              : 73.7%
 [K] MATRIX SIZE             : 3.5 MB
 [M] MATRIX SIZE             : 930.5 KB
 OPTIMUM RAM NEEDED          : 1.6 MB
 FREE SCRATCH SPACE          : 247.4 GB

 Reducing 4560 Equations (Using 3.5 MB RAM)...
```

MAXIMUM PIVOT : 3.800067E+09 (Node 20 DZ)
MINIMUM PIVOT : 3.514328E+03 (Node 173 RY)

NODAL DISPLACEMENT COMPONENTS USED IN STARTING VECTOR

295[DX]	174[DX]	173[DX]	294[DX]	135[DX]	265[DX]
147[DX]	274[DX]	146[DX]	273[DX]	161[DX]	163[DX]
157[DX]	286[DX]	258[DX]			

SUBSPACE ITERATION
Natural Frequencies : 10
Mass Degrees of Freedom : 4332
Subspace Dimension : 16
Convergence Tolerance : 1.000000E-05
Frequency Shift : 0.000000E+00 Hz
THE FIRST 10 EIGENVALUES HAVE CONVERGED

FINAL FREQUENCY RESULTS

Mode	Eigenvalue	Frequency (rad/s)	Frequency (Hz)
1	2.92215455E+05	5.40569566E+02	8.60343185E+01
2	4.22136892E+05	6.49720626E+02	1.03406249E+02
3	1.77783432E+06	1.33335454E+03	2.12209966E+02
4	2.36136288E+06	1.53667267E+03	2.44569051E+02
5	4.18990692E+06	2.04692621E+03	3.25778425E+02
6	8.05259750E+06	2.83770990E+03	4.51635558E+02
7	8.52003168E+06	2.91890933E+03	4.64558849E+02
8	9.95262747E+06	3.15477851E+03	5.02098595E+02
9	1.15626993E+07	3.40039693E+03	5.41189980E+02
10	1.40880592E+07	3.75340635E+03	5.97373174E+02

Mass Participation Factors and Effective Modal Damping...

TOTAL MASS (MX,MY,MZ) : (2.470003E+01, 2.470003E+01, 2.470003E+01)

MODE PARTICIPATION FOR TRANSLATIONAL EXCITATION

Mode	Frequency (Hz)	Modal Mass (Eng)	Modal Stiff (Eng)	PF-X (%)	PF-Y (%)	PF-Z (%)
1	8.6034E+01	5.7673E+00	1.6853E+06	62.505	0.000	0.000
2	1.0341E+02	5.6041E+00	2.3657E+06	0.000	0.000	0.000
3	2.1221E+02	6.8090E+00	1.2105E+07	0.000	65.350	0.000
4	2.4457E+02	3.7401E+00	8.8317E+06	0.053	0.000	0.000
5	3.2578E+02	3.6268E+00	1.5196E+07	5.485	0.000	0.000
6	4.5164E+02	4.0405E+00	3.2537E+07	0.000	0.000	0.000
7	4.6456E+02	2.7009E+00	2.3012E+07	0.312	0.000	0.000
8	5.0210E+02	2.1197E+00	2.1097E+07	0.000	0.000	0.000
9	5.4119E+02	6.3114E+00	7.2977E+07	15.160	0.000	0.000
10	5.9737E+02	6.9852E+00	9.8408E+07	0.000	0.000	0.000
TOTAL TRANSLATIONAL MASS PARTICIPATION FACTORS				83.516	65.350	0.000

MODE PARTICIPATION FOR ROTATIONAL EXCITATION

Mode	Frequency (Hz)	Modal Mass (Eng)	Modal Stiff (Eng)	PF-RX (%)	PF-RY (%)	PF-RZ (%)
1	8.6034E+01	5.7673E+00	1.6853E+06	0.000	97.042	0.000
2	1.0341E+02	5.6041E+00	2.3657E+06	0.000	0.000	63.467
3	2.1221E+02	6.8090E+00	1.2105E+07	98.344	0.000	0.000
4	2.4457E+02	3.7401E+00	8.8317E+06	0.000	0.061	0.000
5	3.2578E+02	3.6268E+00	1.5196E+07	0.000	0.677	0.000

```
   6    4.5164E+02    4.0405E+00    3.2537E+07     0.000     0.000     1.192
   7    4.6456E+02    2.7009E+00    2.3012E+07     0.000     0.135     0.000
   8    5.0210E+02    2.1197E+00    2.1097E+07     0.000     0.000     7.391
   9    5.4119E+02    6.3114E+00    7.2977E+07     0.000     1.691     0.000
  10    5.9737E+02    6.9852E+00    9.8408E+07     0.000     0.000    10.011
----------------------------------------------------------------------------
TOTAL ROTATIONAL MASS PARTICIPATION FACTORS       98.344    99.606    82.061

TOTAL CPU TIME              : 2.120 Seconds ( 0:00:02)
```

FINAL RESULTS

Mode	Automesh (Hz)	Manual mesh (Hz)
1	86.15	86.03
2	103.53	103.41
3	212.19	212.21
4	244.15	244.57
5	325.20	325.78
6	450.20	451.64
7	464.01	464.56
8	501.83	502.10
9	542.66	541.19
10	597.32	597.37

APPENDIX III: RESULTS FEA BEAM-ELEMENTS

203X133X25 BEAM ELEMENTS – WEB MOUNT – FULL ENDPLATE

STRAND7 RESULTS

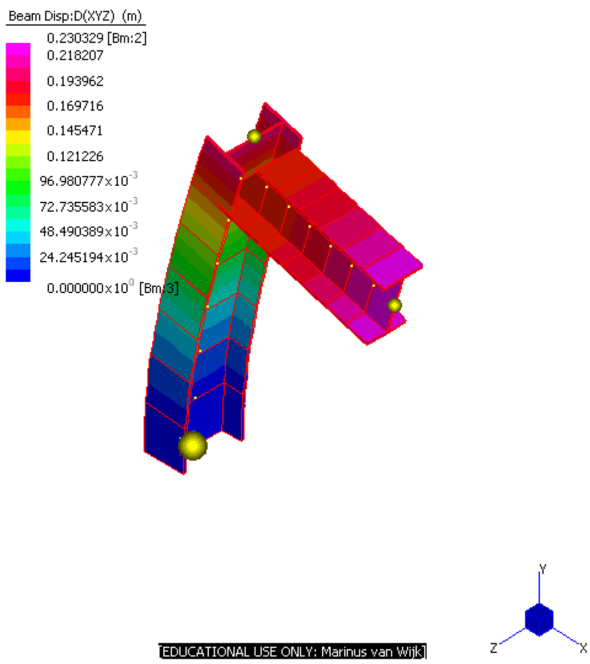
```
___________________________________________
 Solution commenced on 23/10/2014 at 10:22:32

 Strand7 [2.4.6][Solver Build: 24131212] (32-Bit) [EDUCATIONAL USE ONLY]

 ANALYSIS TYPE              : NATURAL FREQUENCY

 COMPUTER NAME              : pbapdesk001
 USER LOGON NAME            : marinusvanwijk
 CPU                        : Intel(R) Pentium(R) 4 CPU 2.40GHz
 USABLE PHYSICAL MEMORY     : 4.0 GB
 USABLE VIRTUAL MEMORY      : 3.0 GB

 MODEL FILE                  : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\203x133x25 Beam
Elements\Full Endplate\203x133x25 Beam - Full Endplate - test.st7"
 RESULT FILE                 : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\203x133x25 Beam
Elements\Full Endplate\203x133x25 Beam - Full Endplate - test.NFA"
 SCRATCH PATH               : "C:\users\crossover\Strand7\Tmp\"

 TOTALS
 Nodes                      : 16
 Beams                      : 14
 Plates                     :  0
 Bricks                     :  0
 Links                      :  1

 SOLVER UNITS
 Length                     : m
 Mass                       : kg
 Force                      : N
 Stress                     : Pa

 FREEDOM CASE               : "Freedom Case 1"

 MASS MATRIX OPTION
 Beam Elements              : Consistent
 Global Matrix              : Full
 Non-Structural Mass        : Added (from the following cases)
                            : "Load Case 1"

 STORAGE SCHEME             : Sparse
 SORTING METHOD             : AMD

 NUMBER OF EQUATIONS        : 84
 MATRIX FILL-IN             : 0.0%
 [K] MATRIX SIZE            : 6.5 KB
 [M] MATRIX SIZE            : 6.5 KB
 OPTIMUM RAM NEEDED         : 64.0 KB
 FREE SCRATCH SPACE         : 240.9 GB
```

```
Reducing 84 Equations (Using 64.0 KB RAM)...

MAXIMUM PIVOT              : 9.026956E+09 (Node 2 DX)
MINIMUM PIVOT              : 5.203433E+03 (Node 16 RY)

NODAL DISPLACEMENT COMPONENTS USED IN STARTING VECTOR
      9[RY]        7[RY]        6[RY]        5[RY]        8[RY]        4[RX]
     16[RX]       11[RX]       12[RX]       13[RX]       15[RX]       14[RX]
      2[RY]        2[DZ]       10[RY]

SUBSPACE ITERATION
Natural Frequencies        : 10
Mass Degrees of Freedom    : 84
Subspace Dimension         : 16
Convergence Tolerance      : 1.000000E-05
Frequency Shift            : 0.000000E+00 Hz

[ITERATIONS REMOVED IN APPENDIX]

THE FIRST 10 EIGENVALUES HAVE CONVERGED

FINAL FREQUENCY RESULTS
Mode        Eigenvalue       Frequency (rad/s)    Frequency (Hz)
   1      3.59567885E+03      5.99639796E+01      9.54356376E+00
   2      7.68422045E+04      2.77204265E+02      4.41184290E+01
   3      2.14077765E+05      4.62685385E+02      7.36386661E+01
   4      2.94900187E+05      5.43047132E+02      8.64286353E+01
   5      8.41451362E+05      9.17306580E+02      1.45993877E+02
   6      1.24927496E+06      1.11770969E+03      1.77889023E+02
   7      1.38205413E+06      1.17560798E+03      1.87103822E+02
   8      2.00120750E+06      1.41464041E+03      2.25147015E+02
   9      3.04846525E+06      1.74598547E+03      2.77882218E+02
  10      5.85045234E+06      2.41877083E+03      3.84959334E+02

Mass Participation Factors and Effective Modal Damping...

TOTAL MASS (MX,MY,MZ)      : ( 4.283193E+01, 4.278317E+01, 4.278795E+01)

MODE PARTICIPATION FOR TRANSLATIONAL EXCITATION
Mode    Frequency     Modal Mass     Modal Stiff    Modal Damp    PF-X      PF-Y      PF-Z
           (Hz)          (Eng)          (Eng)         (Ratio)      (%)       (%)       (%)
   1    9.5436E+00    5.7964E+00    2.0842E+04     0.40000     0.000     0.000    22.877
   2    4.4118E+01    2.3860E+01    1.8335E+06     0.40000    69.641     2.988     0.000
   3    7.3639E+01    1.0538E+04    2.2560E+09     0.40000     0.000     0.000     0.062
   4    8.6429E+01    2.5597E+02    7.5487E+07     0.40000     0.000     0.000     1.479
   5    1.4599E+02    1.5301E+01    1.2875E+07     0.40000     0.000     0.000    56.893
   6    1.7789E+02    4.9013E+02    6.1231E+08     0.40000     0.000     0.000     0.759
   7    1.8710E+02    7.3637E+00    1.0177E+07     0.40000    16.989    19.561     0.000
   8    2.2515E+02    6.5057E+03    1.3019E+10     0.40000     0.000     0.000     0.198
   9    2.7788E+02    2.3336E+03    7.1139E+09     0.40000     0.000     0.000     0.011
  10    3.8496E+02    1.8160E+05    1.0624E+12     0.40000     0.000     0.000     0.000
-----------------------------------------------------------------------------------------
TOTAL TRANSLATIONAL MASS PARTICIPATION FACTORS               86.630    22.549    82.280

MODE PARTICIPATION FOR ROTATIONAL EXCITATION
Mode    Frequency     Modal Mass     Modal Stiff     PF-RX     PF-RY     PF-RZ
           (Hz)          (Eng)          (Eng)         (%)       (%)       (%)
   1    9.5436E+00    5.7964E+00    2.0842E+04     42.103    97.394     0.000
```

```
   2    4.4118E+01    2.3860E+01    1.8335E+06     0.000     0.000    72.406
   3    7.3639E+01    1.0538E+04    2.2560E+09     0.758     0.000     0.000
   4    8.6429E+01    2.5597E+02    7.5487E+07     1.843     2.008     0.000
   5    1.4599E+02    1.5301E+01    1.2875E+07    30.817     0.024     0.000
   6    1.7789E+02    4.9013E+02    6.1231E+08     0.180     0.393     0.000
   7    1.8710E+02    7.3637E+00    1.0177E+07     0.000     0.000     0.595
   8    2.2515E+02    6.5057E+03    1.3019E+10     0.008     0.000     0.000
   9    2.7788E+02    2.3336E+03    7.1139E+09     0.001     0.106     0.000
  10    3.8496E+02    1.8160E+05    1.0624E+12     0.000     0.027     0.000
----------------------------------------------------------------------------
TOTAL ROTATIONAL MASS PARTICIPATION FACTORS       75.710    99.953    73.001

COUNTING MODES IN RANGE     :   9.16814799E+00 to  3.85334750E+02

Reducing 84 Equations (Using 64.0 KB RAM)...

Reducing 84 Equations (Using 64.0 KB RAM)...

STURM CHECK RESULTS
There is no natural frequency below 9.168E+00 Hz.
There are 10 frequencies below 3.853E+02 Hz.
All eigenvalues are found.

TOTAL CPU TIME              : 1.210 Seconds ( 0:00:01)

Solution completed on 23/10/2014 at 10:22:33
Solution time: 3 Seconds

SUMMARY OF MESSAGES
Number of Notes     : 0
Number of Warnings : 0
Number of Errors    : 0
```

PICTURES OF THE RESULTS

MODE 2 – 44.118 HZ



203X133X25 BEAM ELEMENTS – WEB MOUNT – HALF ENDPLATE

STRAND7 RESULTS

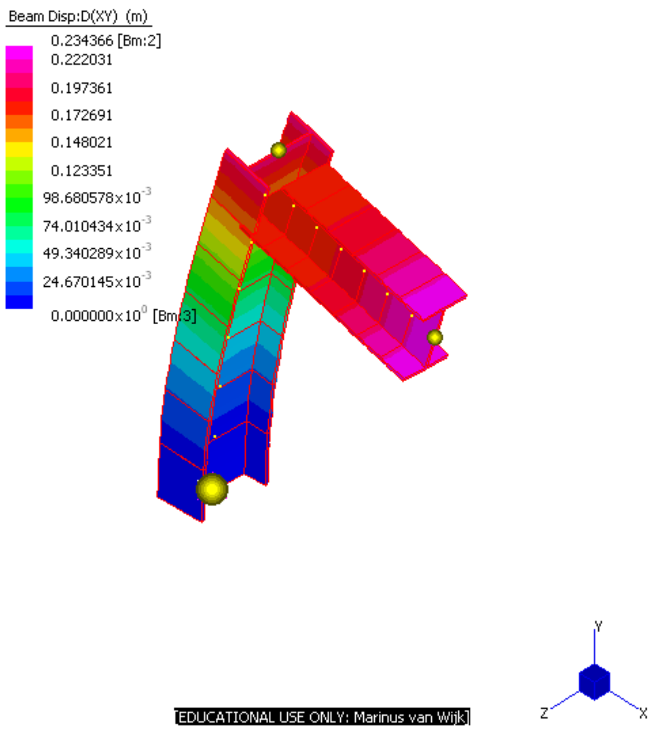
```
___________________________________________
 Solution commenced on 21/10/2014 at 13:22:33

 Strand7 [2.4.6][Solver Build: 24131212] (32-Bit) [EDUCATIONAL USE ONLY]

 ANALYSIS TYPE               : NATURAL FREQUENCY

 COMPUTER NAME               : pbapdesk001
 USER LOGON NAME             : marinusvanwijk
 CPU                         : Intel(R) Pentium(R) 4 CPU 2.40GHz
 USABLE PHYSICAL MEMORY      : 4.0 GB
 USABLE VIRTUAL MEMORY       : 3.0 GB

 MODEL FILE                  : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\203x133x25 Beam
Elements\Half Endplate\203x133x25 Beam - Half Endplate - test.st7"
 RESULT FILE                 : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\203x133x25 Beam
Elements\Half Endplate\203x133x25 Beam - Half Endplate - test.NFA"
 SCRATCH PATH                : "C:\users\crossover\Strand7\Tmp\"

 TOTALS
 Nodes                       : 17
 Beams                       : 15
 Plates                      :  0
 Bricks                      :  0
 Links                       :  1

 SOLVER UNITS
 Length                      : m
 Mass                        : kg
 Force                       : N
 Stress                      : Pa

 FREEDOM CASE                : "Freedom Case 1"

 MASS MATRIX OPTION
 Beam Elements               : Consistent
 Global Matrix               : Full
 Non-Structural Mass         : Added (from the following cases)
                             : "Load Case 1"

 STORAGE SCHEME              : Sparse
 SORTING METHOD              : AMD

 NUMBER OF EQUATIONS         : 90
 MATRIX FILL-IN              : 0.0%
 [K] MATRIX SIZE             : 7.0 KB
 [M] MATRIX SIZE             : 7.0 KB
 OPTIMUM RAM NEEDED          : 64.0 KB
 FREE SCRATCH SPACE          : 241.2 GB

 Reducing 90 Equations (Using 64.0 KB RAM)...

 MAXIMUM PIVOT               : 1.902154E+11 (Node 2 DY)
 MINIMUM PIVOT               : 4.585314E+03 (Node 16 RY)
```

```
NODAL DISPLACEMENT COMPONENTS USED IN STARTING VECTOR
      9[RY]        7[RY]        6[RY]        5[RY]        8[RY]        2[RY]
     10[RY]        4[RX]       16[RX]       11[RX]       12[RX]       13[RX]
     15[RX]       14[RX]        3[RX]

SUBSPACE ITERATION
Natural Frequencies         : 10
Mass Degrees of Freedom     : 90
Subspace Dimension          : 16
Convergence Tolerance       : 1.000000E-05
Frequency Shift             : 0.000000E+00 Hz

[ITERATIONS REMOVED IN APPENDIX]

THE FIRST 10 EIGENVALUES HAVE CONVERGED

FINAL FREQUENCY RESULTS
Mode        Eigenvalue       Frequency (rad/s)    Frequency (Hz)
   1      2.95624258E+03      5.43713397E+01      8.65346747E+00
   2      7.63281524E+04      2.76275501E+02      4.39706116E+01
   3      1.60269135E+05      4.00336277E+02      6.37154974E+01
   4      2.25135686E+05      4.74484653E+02      7.55165780E+01
   5      8.12809792E+05      9.01559644E+02      1.43487674E+02
   6      8.42103439E+05      9.17661941E+02      1.46050434E+02
   7      9.37691848E+05      9.68344901E+02      1.54116878E+02
   8      1.52903177E+06      1.23654024E+03      1.96801492E+02
   9      1.75943213E+06      1.32643587E+03      2.11108826E+02
  10      3.26087244E+06      1.80578859E+03      2.87400181E+02

Mass Participation Factors and Effective Modal Damping...

TOTAL MASS (MX,MY,MZ)       : ( 4.248090E+01, 4.243214E+01, 4.243692E+01)

MODE PARTICIPATION FOR TRANSLATIONAL EXCITATION
Mode    Frequency     Modal Mass    Modal Stiff     PF-X      PF-Y      PF-Z
           (Hz)          (Eng)         (Eng)         (%)       (%)       (%)
   1    8.6535E+00    6.1206E+00    1.8094E+04     0.000     0.000    24.527
   2    4.3971E+01    2.4164E+01    1.8444E+06    67.599     3.737     0.000
   3    6.3715E+01    2.0099E+04    3.2213E+09     0.000     0.000     0.029
   4    7.5517E+01    2.0162E+02    4.5392E+07     0.000     0.000     1.386
   5    1.4349E+02    4.6610E+01    3.7885E+07     0.000     0.000    20.000
   6    1.4605E+02    7.5441E+00    6.3529E+06    16.625    20.718     0.000
   7    1.5412E+02    2.0264E+01    1.9001E+07     0.000     0.000    35.945
   8    1.9680E+02    3.8766E+03    5.9274E+09     0.000     0.000     0.299
   9    2.1111E+02    9.2896E+02    1.6344E+09     0.000     0.000     0.080
  10    2.8740E+02    1.2097E+04    3.9448E+10     0.000     0.000     0.056
----------------------------------------------------------------------------
TOTAL TRANSLATIONAL MASS PARTICIPATION FACTORS    84.224    24.455    82.321

MODE PARTICIPATION FOR ROTATIONAL EXCITATION
Mode    Frequency     Modal Mass    Modal Stiff     PF-RX     PF-RY     PF-RZ
           (Hz)          (Eng)         (Eng)         (%)       (%)       (%)
   1    8.6535E+00    6.1206E+00    1.8094E+04    35.842    97.091     0.000
   2    4.3971E+01    2.4164E+01    1.8444E+06     0.000     0.000    93.486
   3    6.3715E+01    2.0099E+04    3.2213E+09     0.663     0.000     0.000
   4    7.5517E+01    2.0162E+02    4.5392E+07     1.832     2.200     0.000
   5    1.4349E+02    4.6610E+01    3.7885E+07    21.106     0.321     0.000
```

```
  6    1.4605E+02    7.5441E+00    6.3529E+06     0.000     0.000     2.227
  7    1.5412E+02    2.0264E+01    1.9001E+07    36.167     0.124     0.000
  8    1.9680E+02    3.8766E+03    5.9274E+09     0.125     0.000     0.000
  9    2.1111E+02    9.2896E+02    1.6344E+09     0.059     0.166     0.000
 10    2.8740E+02    1.2097E+04    3.9448E+10     0.075     0.027     0.000
----------------------------------------------------------------------------
TOTAL ROTATIONAL MASS PARTICIPATION FACTORS       95.868    99.929    95.713

COUNTING MODES IN RANGE     :   8.37472076E+00 to  2.87678927E+02

Reducing 90 Equations (Using 64.0 KB RAM)...

Reducing 90 Equations (Using 64.0 KB RAM)...

STURM CHECK RESULTS
There is no natural frequency below 8.375E+00 Hz.
There are 10 frequencies below 2.877E+02 Hz.
All eigenvalues are found.

TOTAL CPU TIME              : 1.350 Seconds ( 0:00:01)

Solution completed on 21/10/2014 at 13:22:34
Solution time: 3 Seconds

SUMMARY OF MESSAGES
Number of Notes    : 0
Number of Warnings : 0
Number of Errors   : 0
```

Pictures of the results

Mode 2 – 43.971 Hz



203X133X25 BEAM ELEMENTS – FLANGE MOUNT – FULL ENDPLATE

STRAND7 RESULTS

```
Solution commenced on 21/10/2014 at 13:25:54

Strand7 [2.4.6][Solver Build: 24131212] (32-Bit) [EDUCATIONAL USE ONLY]

ANALYSIS TYPE               : NATURAL FREQUENCY

COMPUTER NAME               : pbapdesk001
USER LOGON NAME             : marinusvanwijk
CPU                         : Intel(R) Pentium(R) 4 CPU 2.40GHz
USABLE PHYSICAL MEMORY      : 4.0 GB
USABLE VIRTUAL MEMORY       : 3.0 GB

MODEL FILE                      : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\S-203x133x25
Beam Elements\Full Endplate\S-203x133x25 Beam - Full Endplate - test.st7"
RESULT FILE                     : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\S-203x133x25
Beam Elements\Full Endplate\S-203x133x25 Beam - Full Endplate - test.NFA"
SCRATCH PATH                : "C:\users\crossover\Strand7\Tmp\"

TOTALS
Nodes                       : 16
Beams                       : 14
Plates                      :  0
Bricks                      :  0
Links                       :  1

SOLVER UNITS
Length                      : m
Mass                        : kg
Force                       : N
Stress                      : Pa

FREEDOM CASE                : "Freedom Case 1"

MASS MATRIX OPTION
Beam Elements               : Consistent
Global Matrix               : Full
Non-Structural Mass         : Added (from the following cases)
                            : "Load Case 1"

STORAGE SCHEME              : Sparse
SORTING METHOD              : AMD

NUMBER OF EQUATIONS         : 84
MATRIX FILL-IN              : 0.0%
[K] MATRIX SIZE             : 6.5 KB
[M] MATRIX SIZE             : 6.5 KB
OPTIMUM RAM NEEDED          : 64.0 KB
FREE SCRATCH SPACE          : 241.2 GB

Reducing 84 Equations (Using 64.0 KB RAM)...

MAXIMUM PIVOT               : 9.062292E+09 (Node 2 DX)
MINIMUM PIVOT               : 5.203433E+03 (Node 16 RY)
```

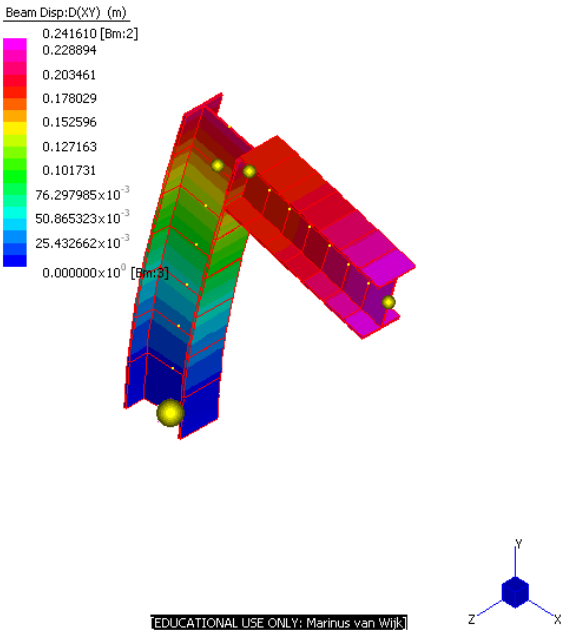
```
NODAL DISPLACEMENT COMPONENTS USED IN STARTING VECTOR
      9[RY]        7[RY]        6[RY]        5[RY]        8[RY]       12[RX]
     15[RX]       11[RX]       13[RX]       14[RX]       16[RX]        4[RX]
      2[RY]        2[DZ]       10[RY]

SUBSPACE ITERATION
Natural Frequencies         : 10
Mass Degrees of Freedom     : 84
Subspace Dimension          : 16
Convergence Tolerance       : 1.000000E-05
Frequency Shift             : 0.000000E+00 Hz

ITERATION 1
Current Frequencies (Hz)
 9.49171E+00  6.22881E+01  7.52025E+01  8.84204E+01  1.37400E+02  1.77696E+02
 2.24550E+02  2.78069E+02  3.84965E+02  3.86352E+02  4.62287E+02  5.26442E+02
 5.66391E+02  7.67053E+02  9.71269E+02  1.11674E+03

[ITERATIONS REMOVED IN APPENDIX]

THE FIRST 10 EIGENVALUES HAVE CONVERGED

FINAL FREQUENCY RESULTS
Mode        Eigenvalue       Frequency (rad/s)    Frequency (Hz)
   1      3.55301837E+03      5.96072006E+01      9.48678063E+00
   2      1.51106328E+05      3.88723974E+02      6.18673419E+01
   3      2.22574860E+05      4.71778402E+02      7.50858647E+01
   4      3.07467447E+05      5.54497473E+02      8.82510137E+01
   5      5.45440828E+05      7.38539659E+02      1.17542237E+02
   6      1.24359752E+06      1.11516704E+03      1.77484346E+02
   7      1.98719874E+06      1.40968037E+03      2.24357599E+02
   8      2.98282258E+06      1.72708499E+03      2.74874114E+02
   9      3.04810578E+06      1.74588252E+03      2.77865833E+02
  10      5.83431692E+06      2.41543307E+03      3.84428113E+02

Mass Participation Factors and Effective Modal Damping...

TOTAL MASS (MX,MY,MZ)       : ( 4.013495E+01, 4.013017E+01, 4.017893E+01)

MODE PARTICIPATION FOR TRANSLATIONAL EXCITATION
Mode    Frequency     Modal Mass    Modal Stiff    PF-X      PF-Y      PF-Z
           (Hz)          (Eng)         (Eng)        (%)       (%)       (%)
   1    9.4868E+00    5.8295E+00    2.0712E+04     0.000     0.000    25.443
   2    6.1867E+01    1.4132E+01    2.1354E+06     0.000     0.000    46.148
   3    7.5086E+01    1.1480E+02    2.5551E+07     0.000     0.000     6.839
   4    8.8251E+01    1.2899E+02    3.9659E+07     0.000     0.000     1.639
   5    1.1754E+02    2.2294E+01    1.2160E+07    68.546     3.656     0.000
   6    1.7748E+02    6.7319E+02    8.3717E+08     0.000     0.000     0.022
   7    2.2436E+02    3.3786E+03    6.7140E+09     0.000     0.000     0.213
   8    2.7487E+02    6.9544E+00    2.0744E+07    15.358    24.155     0.000
   9    2.7787E+02    2.4232E+03    7.3861E+09     0.000     0.000     0.005
  10    3.8443E+02    4.1751E+02    2.4359E+09     0.000     0.000     0.614
---------------------------------------------------------------------------
TOTAL TRANSLATIONAL MASS PARTICIPATION FACTORS    83.904    27.811    80.921

MODE PARTICIPATION FOR ROTATIONAL EXCITATION
Mode    Frequency     Modal Mass    Modal Stiff    PF-RX     PF-RY     PF-RZ
```

	(Hz)	(Eng)	(Eng)	(%)	(%)	(%)
1	9.4868E+00	5.8295E+00	2.0712E+04	45.634	97.738	0.000
2	6.1867E+01	1.4132E+01	2.1354E+06	32.690	0.014	0.000
3	7.5086E+01	1.1480E+02	2.5551E+07	2.200	0.000	0.000
4	8.8251E+01	1.2899E+02	3.9659E+07	0.293	0.810	0.000
5	1.1754E+02	2.2294E+01	1.2160E+07	0.000	0.000	81.253
6	1.7748E+02	6.7319E+02	8.3717E+08	0.008	0.389	0.000
7	2.2436E+02	3.3786E+03	6.7140E+09	0.007	0.000	0.000
8	2.7487E+02	6.9544E+00	2.0744E+07	0.000	0.000	0.008
9	2.7787E+02	2.4232E+03	7.3861E+09	0.003	0.041	0.000
10	3.8443E+02	4.1751E+02	2.4359E+09	0.102	0.001	0.000
TOTAL ROTATIONAL MASS PARTICIPATION FACTORS				80.938	98.993	81.262

```
COUNTING MODES IN RANGE     :   9.11183929E+00 to  3.84803054E+02

Reducing 84 Equations (Using 64.0 KB RAM)...

Reducing 84 Equations (Using 64.0 KB RAM)...

STURM CHECK RESULTS
There is no natural frequency below 9.112E+00 Hz.
There are 10 frequencies below 3.848E+02 Hz.
All eigenvalues are found.

TOTAL CPU TIME              : 1.240 Seconds ( 0:00:01)

Solution completed on 21/10/2014 at 13:25:55
Solution time: 3 Seconds

SUMMARY OF MESSAGES
Number of Notes     : 0
Number of Warnings : 0
Number of Errors    : 0
```

Pictures of the results

Mode 5 – 117.54 Hz



203X133X25 BEAM ELEMENTS – FLANGE MOUNT – HALF ENDPLATE

STRAND7 RESULTS

```
Solution commenced on 21/10/2014 at 13:34:06

Strand7 [2.4.6][Solver Build: 24131212] (32-Bit) [EDUCATIONAL USE ONLY]

ANALYSIS TYPE               : NATURAL FREQUENCY

COMPUTER NAME               : pbapdesk001
USER LOGON NAME             : marinusvanwijk
CPU                         : Intel(R) Pentium(R) 4 CPU 2.40GHz
USABLE PHYSICAL MEMORY      : 4.0 GB
USABLE VIRTUAL MEMORY       : 3.0 GB

MODEL FILE                      : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\S-203x133x25
Beam Elements\Half Endplate\S-203x133x25 Beam - Half Endplate - test.st7"
RESULT FILE                     : "C:\users\crossover\Desktop\My Mac Desktop\Strand7\ModelPBA\S-203x133x25
Beam Elements\Half Endplate\S-203x133x25 Beam - Half Endplate - test.NFA"
SCRATCH PATH                : "C:\users\crossover\Strand7\Tmp\"

TOTALS
Nodes                       : 17
Beams                       : 15
Plates                      :  0
Bricks                      :  0
Links                       :  1

SOLVER UNITS
Length                      : m
Mass                        : kg
Force                       : N
Stress                      : Pa

FREEDOM CASE                : "Freedom Case 1"

MASS MATRIX OPTION
Beam Elements               : Consistent
Global Matrix               : Full
Non-Structural Mass         : Added (from the following cases)
                            : "Load Case 1"

STORAGE SCHEME              : Sparse
SORTING METHOD              : AMD

NUMBER OF EQUATIONS         : 90
MATRIX FILL-IN              : 0.0%
[K] MATRIX SIZE             : 7.0 KB
[M] MATRIX SIZE             : 7.0 KB
OPTIMUM RAM NEEDED          : 64.0 KB
FREE SCRATCH SPACE          : 241.2 GB

Reducing 90 Equations (Using 64.0 KB RAM)...

MAXIMUM PIVOT               : 9.041718E+09 (Node 3 DX)
MINIMUM PIVOT               : 5.198810E+03 (Node 16 RY)
```

```
NODAL DISPLACEMENT COMPONENTS USED IN STARTING VECTOR
      9[RY]        7[RY]        6[RY]        5[RY]        8[RY]        3[RX]
     12[RX]       15[RX]       11[RX]       13[RX]       14[RX]       16[RX]
      4[RX]        2[RY]       10[RY]

SUBSPACE ITERATION
Natural Frequencies         : 10
Mass Degrees of Freedom     : 90
Subspace Dimension          : 16
Convergence Tolerance       : 1.000000E-05
Frequency Shift             : 0.000000E+00 Hz

[ITERATIONS REMOVED IN APPENDIX]

THE FIRST 10 EIGENVALUES HAVE CONVERGED

FINAL FREQUENCY RESULTS
Mode        Eigenvalue       Frequency (rad/s)    Frequency (Hz)
   1      2.03090416E+03      4.50655541E+01      7.17240569E+00
   2      1.37506162E+05      3.70818233E+02      5.90175548E+01
   3      1.93321241E+05      4.39683115E+02      6.99777411E+01
   4      3.03788802E+05      5.51170393E+02      8.77214925E+01
   5      4.80067622E+05      6.92869123E+02      1.10273546E+02
   6      1.23894813E+06      1.11308047E+03      1.77152258E+02
   7      1.34503354E+06      1.15975581E+03      1.84580870E+02
   8      1.92703572E+06      1.38817712E+03      2.20935251E+02
   9      3.04448420E+06      1.74484503E+03      2.77700712E+02
  10      3.92553854E+06      1.98129719E+03      3.15333241E+02

Mass Participation Factors and Effective Modal Damping...

TOTAL MASS (MX,MY,MZ)       : ( 4.016920E+01, 4.016442E+01, 4.021318E+01)

MODE PARTICIPATION FOR TRANSLATIONAL EXCITATION
Mode    Frequency     Modal Mass     Modal Stiff     PF-X      PF-Y      PF-Z
          (Hz)          (Eng)          (Eng)         (%)       (%)       (%)
   1    7.1724E+00    6.9654E+00    1.4146E+04     0.000     0.000    31.182
   2    5.9018E+01    6.9840E+01    9.6034E+06     0.000     0.000     6.700
   3    6.9978E+01    1.3049E+01    2.5226E+06     0.000     0.000    40.172
   4    8.7721E+01    1.1520E+02    3.4998E+07     0.000     0.000     2.141
   5    1.1027E+02    2.3928E+01    1.1487E+07    56.522     8.983     0.000
   6    1.7715E+02    1.1671E+03    1.4459E+09     0.000     0.000     0.016
   7    1.8458E+02    3.7508E+03    5.0450E+09     0.000     0.000     0.227
   8    2.2094E+02    9.8656E+00    1.9011E+07    27.163    23.144     0.000
   9    2.7770E+02    3.8031E+03    1.1579E+10     0.000     0.000     0.004
  10    3.1533E+02    1.7387E+03    6.8255E+09     0.000     0.000     0.347
---------------------------------------------------------------------------
TOTAL TRANSLATIONAL MASS PARTICIPATION FACTORS    83.685    32.128    80.789

MODE PARTICIPATION FOR ROTATIONAL EXCITATION
Mode    Frequency     Modal Mass     Modal Stiff     PF-RX     PF-RY     PF-RZ
          (Hz)          (Eng)          (Eng)         (%)       (%)       (%)
   1    7.1724E+00    6.9654E+00    1.4146E+04    50.660    98.351     0.000
   2    5.9018E+01    6.9840E+01    9.6034E+06     8.929     0.021     0.000
   3    6.9978E+01    1.3049E+01    2.5226E+06    29.101     0.175     0.000
   4    8.7721E+01    1.1520E+02    3.4998E+07     1.035     1.094     0.000
   5    1.1027E+02    2.3928E+01    1.1487E+07     0.000     0.000    87.043
```

6	1.7715E+02	1.1671E+03	1.4459E+09	0.000	0.240	0.000
7	1.8458E+02	3.7508E+03	5.0450E+09	0.016	0.000	0.000
8	2.2094E+02	9.8656E+00	1.9011E+07	0.000	0.000	3.940
9	2.7770E+02	3.8031E+03	1.1579E+10	0.000	0.066	0.000
10	3.1533E+02	1.7387E+03	6.8255E+09	0.095	0.059	0.000

```
----------------------------------------------------------------------------
TOTAL ROTATIONAL MASS PARTICIPATION FACTORS       89.836   100.006    90.982

COUNTING MODES IN RANGE     :   6.86424486E+00 to  3.15641402E+02

Reducing 90 Equations (Using 64.0 KB RAM)...

Reducing 90 Equations (Using 64.0 KB RAM)...

STURM CHECK RESULTS
There is no natural frequency below 6.864E+00 Hz.
There are 10 frequencies below 3.156E+02 Hz.
All eigenvalues are found.

TOTAL CPU TIME              : 1.330 Seconds ( 0:00:01)

Solution completed on 21/10/2014 at 13:34:07
Solution time: 5 Seconds

SUMMARY OF MESSAGES
Number of Notes     : 0
Number of Warnings  : 0
Number of Errors    : 0
```

Pictures of the results

Mode 5 – 110.27 Hz