OPTIMAL PATH PLANNING FOR REARWARD DOCKING OF ARTICULATED VEHICLES

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Abstract

Docking with a tractor semitrailer combination at a distribution site is a stressful and demanding task for the driver. It has to be carried out fast, and there is usually limited space to manoeuvre and to move from forward to rearward motion, where this last phase in the docking manoeuvre has to be carried out very precise to avoid damage. This paper examines the entire docking manoeuvre, consisting of a first phase of approaching the vicinity of the docking gate, followed by the next phase of positioning in such a way that the final reverse motion results in a very accurate (order of centimetres) parking manoeuvre of the trailer towards the docking gate. First, a reference path is determined for the full manoeuvre of forward and rearward motion, followed by bidirectional path following control.

The parameters of the manoeuvre are next varied to examine the impact on the total docking procedure and to find the conditions under which optimal docking is achieved, where docking time is considered as the cost function to be minimized, where a maximum steering angle is assumed. It is shown that the optimal manoeuvre depends strongly on the phase of vehicle forward positioning. Starting this positioning too soon results in a considerably long time before also the trailer is in position to start the reverse docking successfully. Starting it too late means that the trailer axle cuts its reverse trajectory, which results in a long time to correct that. The best strategy consists of a steering history such that the trailer axle is directly pulled to its reverse path by the tractor after which the rearward motion can be started such that this trailer axle follows this reverse path with hardly any deviation. In practical terms, the vehicle starts its positioning manoeuvre when the steering axle is near passing the docking gate. Discussion with truck drivers confirmed that this is indeed the practical approach they usually apply. The accuracy of the total control algorithm (design of the reference path, followed by path tracking control) in the final positioning of rear tractor axle and trailer axle appears to be extremely high with path deviations below 0.01 m.

Keywords: driver support, smart yard, vehicle automation, closed loop performanc

1. Introduction

In general, at distribution centers, tractor-semitrailers have to dock in reverse direction under an angle which may vary from zero to 90 degrees. This docking has to be carried out carefully, to avoid hitting other vehicles, or getting damaged by not positioning accurately at the docking gate, and hitting navigation rail or other type of property damage. Damage may be minor but transport delays may be significant. One prefers to perform the entire manoeuvre (forward entering, followed by reverse parking at the docking gate) as fast as possible, which may be complicated by certain obstacles such as other parked cars or limited available manoeuvring space.
This problem was considered by Karel Kural [5] for HCV's (High Capacity Vehicles, specifically a rigid truck with two central axle trailers), and he derived a bi-directional path following controller for guiding the vehicle successfully through the entire docking process. In his research, the vehicle approaches the distribution centre in a direction parallel to the docking gate at a certain distance of this gate, followed by a low speed forward motion in a way such that the vehicle is positioned in a more or less optimal way for the reverse manoeuvre afterwards.
This approach was based on Dubins curves, i.e. a combination of straight segments and circular arcs as a basis for the planning of the forward reference path, and for the vehicle to follow as good as possible through a path tracking approach. In spite of the fact that the controller brought the vehicle at the docking gate with final lateral and orientation tracking error within acceptable tolerances, the Dubins approach was considered not to reflect sufficiently the actual vehicle transient yaw response. Tracking errors were observed during the forward part of the total manoeuvre, which were considered too large. It was envisaged that the controller performance could be improved for a path planning approach where the final reference path would be closer to the real kinematic vehicle behaviour.

This paper considers the **optimal accurate** docking problem for a tractor-semitrailer combination, where both forward and reverse reference paths are based on kinematic performance of the vehicle at hand. Accurate means that the path planning is such that the tracking error to the (derived) reference paths during the bi-directional docking manoeuvre is within acceptable tolerances, and smaller compared to Dubins approach, especially for the reverse part of the manoeuvre. Optimal means that the total docking manoeuvre is carried out within a small time, in path planning analyses usually referred to as the cost function.

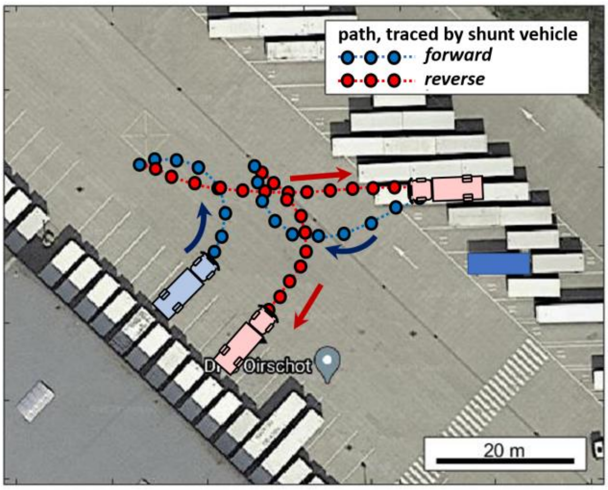

Figure 1.: Docking motion shunt vehicle, DPD site

The research is part of a more extensive research in path planning, where vehicle motion, being either automated or under control of a driver assistance system, is explored for a distribution centre, avoiding static and moving obstacles, and also allowing docking. Figure 1

shows results of an investigation, that was carried out by Manojpriyadharson Kannan [3] of HAN University at the DPD DC in Oirschot, the Netherlands, as part of a large nationwide project CATALYST (Living Lab on connected transport and logistics, see [1]). The dotted lines indicate a so-called shunt vehicle to move away from a docking station, pick up a swap body at a parking place, and moving back to a docking gate to deliver this load unit. Consider the very tight area in which the total maneuver takes place.
This paper is organized as follows. In the next chapter, we discuss the objectives for this research, followed by a description of the research approach in chapter 3. In chapter 4, we include a short discussion in Dubin's curves. Chapters 5 and 6 deal first with the derivation of the reference path consisting of a forward and a rearward part, followed by the bi-directional path tracking analysis leading to the final vehicle path in terms of the tractor front and rear (drive) axle, and the rear trailer axle group center point. Conclusions are discussed in chapter 7.

2. Objectives

The objective of the research is to understand the process of bidirectional path following control, with application in smart docking and driver reverse docking support. Two important aspects of docking at distribution centres are considered:

- The bidirectional controller should, in combination with realistic reference path planning, lead to small errors in tracking these reference paths
- The total vehicle path (forward positioning, reverse docking) should not be larger than necessary to finalize the docking manoeuvre.

In addition, we are interested in the correspondence of the final optimal accurate vehicle path with the experience of truck drivers. We were able to find a very exact estimate of a range for onset of the forward motion (deviating from the motion parallel to the docking gate) to guarantee optimal manoeuvring, which was confirmed by driver experience.

3. Research approach

We assume (low speed) behaviour, both in forward and in reverse direction, with a constant speed V. We have sketched the docking scenario schematically below, as treated in the paper, as well as the expected reverse paths for rear tractor and rear trailer axles. The reverse paths are derived from leaving the docking gate along a straight path, followed by a ramp steer.
Figure 2 shows three phases in the docking manoeuvre:

1. The vehicle starts from a straight line, and moves to a path which corresponds to the reverse docking path for the rear tractor axle. In order to dock in a successful way, we aim to bring all axles close to their reverse docking paths. Phase 1 ends when the rear tractor axle first hits its reverse path in a smooth way which means that the curvature and therefore the steering angle is continuous.
2. Two possible situations may occur. The trailer axle may already have approached its respective reverse docking path very closely when the rear tractor axle has reached point P_2, such that the reverse docking phase can start. Or, this is not yet the case and the tractor has to move further forward beyond point P_2 following its reverse docking

path until the trailer axle is also close enough to its respective reverse docking path (designated as P_3). Note that we have separate reverse paths for tractor front and rear axle and the trailer axle.

3. The final phase is the reverse motion, along the expected reverse path between P_3 and P_4.

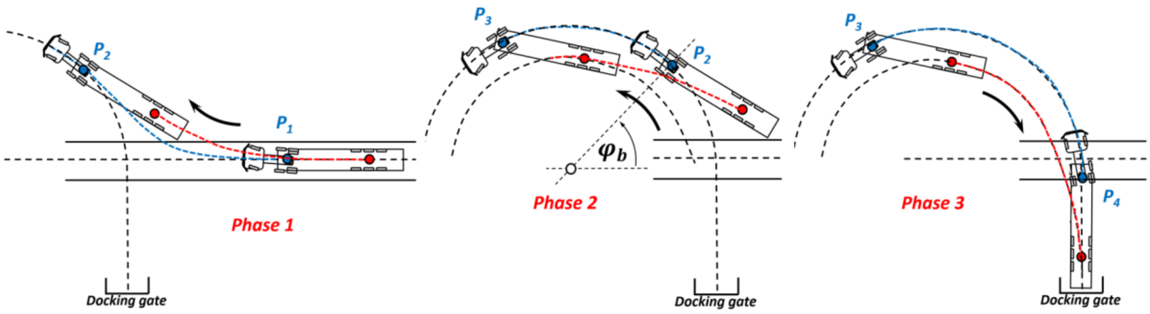

Figure 2.: Layout docking scenario

To find the paths in phases 1 to 3, we shall first determine reference paths, being as close as possible to the final vehicle behaviour. Next we follow a bi-directional path tracking model where the vehicle is following the reference path with some preview distance and (corrective) steering gain, where kinematic conditions are assumed. This means that lateral tyre slip is neglected. In fact, our calculations are based on the kinematic bicycle model, which means that tyres are lumped to single wheels underneath the vehicle at the axle positions, and where roll and pitch are neglected. A kinematic model is quite common in this kind of low speed manoeuvring research on dry road surface conditions, see for example Lavalle [12] and Paden [8]. Evidence that this approach can be considered as an adequate approach was given by Kusumakar [6] who compared a kinematic model of a double articulated longer heavier vehicle (truck-dolly-centre axle trailer) with a multibody model for a speed of 1 m/s. leading to only minor differences. See also Pauwelussen [9], [10].

Kinematic behaviour can be described by the following equations in yaw angles θ_1 and θ_2 for tractor and semi-trailer respectively, and the articulation angle γ:

$$\dot{\theta}_1 = \frac{V}{L_1}.tan(\delta_1) \quad (1)$$

$$\dot{\theta}_2 = \frac{V}{L_2}.sin(\gamma) - \frac{f}{L_2}.\dot{\theta}_1.\cos(\gamma) \quad (2)$$

$$\gamma = \theta_1 - \theta_2 \quad (3)$$

with the various vehicle parameters explained in figure 3. Wheelbases are denoted as L_1 and L_2, the distance between the coupling point and rear axle as f (> 0 if coupling point behind rear axle center point), and path radii as R_r, R and R_f. We take the origin such that the vehicle starts along the line $y = 0$ and the docking gate is situated at $x = 0$. The initial position of the front axle, just before the docking manoeuvre (phase 1) is located at $x_f = -L_c$, and therefore we start with $x_0 = -L_c - L_1$ and $x_r = x_0 - f - L_2$. Figure 3 doesn't match with figure 2 in the sense that the vehicle starts moving to the right. We have drawn that differently in figure 2 because the driver is usually approaching the docking gate from the right-hand side. In that way, he is able to view the entire side of the vehicle combination during the reverse manoeuvre. In this paper, the vehicle is assumed to move in the positive x-direction. We have used the following vehicle data:

L_1:	3.8 [m]	L_2:	7.6 [m]	f:	-0.7 [m]

Table 1.: Vehicle data

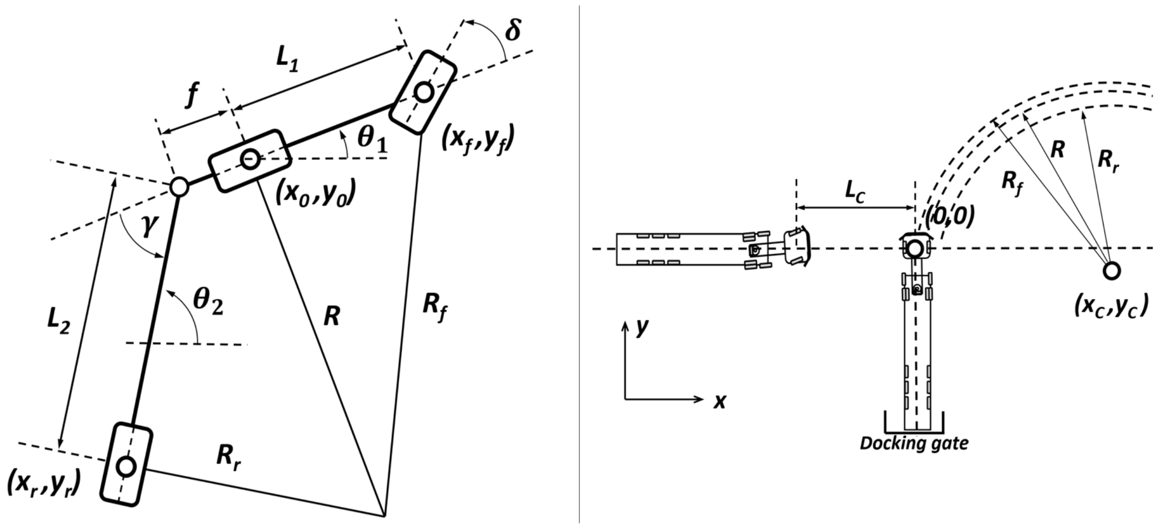

Figure 3.: Schematic layout vehicle model and docking environment

The reference transition between points P_1 and P_2 will be described by a polynomial with continuity in position, orientation and curvature at both ends of the transition. Kinematic behaviour allows the vehicle to continue with constant steering angle along the reverse path following point P_2. Note that the points P_1 and P_2 are free to be chosen as well as the steady state radius while reversing. Different choices will lead to different transitions.

After reaching point P_2, and if the trailer axle is not yet close enough to its respective reverse path, we follow the predefined tractor drive axle reverse path up to the situation that the path error for the trailer axle (with the trailer axle reverse path) is sufficiently small.
The total curve consisting of the transition plus the follow-up along the predefined reverse path is considered as the reference path for the forward motion, to be followed by the vehicle using a path tracking approach. At the end of the path, the reverse manoeuvre is started, which means that the reverse reference path is followed, also using a path tracking approach. The reference path for reverse motion is based on a ramp steer analysis for the tractor-semitrailer, starting at the docking gate and saturating in a circular path with centre (x_C, y_C), with different radii for the three axles.
The reverse path tracking is carried out through a so-called virtual steering angle approach [4,5] at the rear trailer axle. For tracking performance of tractor-semitrailer combinations see also [10]. We will vary the points P_1 and P_2, to find the impact on the position of point P_3.

With P_3 reached earlier, the total vehicle path of the manoeuvre becomes shorter, and therefore more optimal.

4. Docking control using Dubin's curves

Previous research into docking manoeuvres was carried out by Karel Kural, see [4] and [5]. He used an approach where the reference path was built up from circles with constant radius and/or straight lines. This is motivated by the results by Dubin [2] stating that the shortest path for a non-articulated vehicle between nodes on a flat surface is obtained by a combination of circular arcs and straight lines. The radius of an arc is then related to the maximum applicable steering angle. There are arguments why this approach is not the most optimal one to derive the reference path:

(i). A tractor-semitrailer is not able to follow an exact path built up of such elements. It leads to discontinuities in the curvature and therefore in the steering, which is obviously not possible in practice.

(ii). Using these Dubin's curves for a reference path, it was shown in [5] that, for a docking manoeuvre with a rigid truck with two central axle trailers, path tracking would lead to deviations (lateral tracking errors) between vehicle path and reference path in the order of 0.5 m for the forward motion part and 0.1 – 0.15 m for the reverse moving part. Of course, an A-Double is not the same as a tractor-semitrailer, but with the usual narrow space for docking, it is of interest to aim for smaller tracking errors.

On the other hand, comparing Dubin's approach and our approach, they do not seem to be that different, and the present paper suggests an obvious extension of Dubin's approach. The problem of discontinuous curvature can be 'repaired' by determining smooth transitions between the Dubin's elements. In this paper, we do that for a straight line and a circle (discussed in section 5.1), but it is straightforward to add more Dubin's elements and transitions between them.

5. The reference path

As indicated in figure 2, the reference path consists of three parts at most, which will be separately treated in this chapter. Since phase 2 ends at the rearward reference path for the rear tractor axle, it is convenient to discuss the reference path according to phase 3, before we treat phase 2.

5.1. Phase 1 of the forward transition part of the docking process

Finding the transition in phase 1 of the manoeuvre, with the rear tractor axle moving from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ means that we find a path $y = F(x)$ matching position, orientation and curvature at both P_1 and P_2. The reason that we base our transition path on the position of the rear tractor axle is that we have to move from spatial domain to time domain to determine yaw angles from which the rear trailer axle position can be determined. For the tractor axle, we know (assuming kinematic conditions) that the speed direction is along the axle orientation, which is convenient, especially for a constant speed. At the front steering axle, the local speed changes with the steering angle, which makes it more difficult to describe the (front axle) path in the time domain.

Hence, we have six conditions to fulfill,

$$\begin{aligned} F(x_1) = y_1, \quad F(x_2) = y_2 \\ F'(x_1) = f_1, \quad F'(x_2) = f_2 \\ \kappa(x_1) = k_1, \quad \kappa(x_2) = k_2 \end{aligned} \tag{4}$$

for given slopes f_i and curvatures k_i with the curvature given by:

$$\kappa(x) = \frac{F''(x)}{(1+F'(x)^2)^{\frac{3}{2}}} \tag{5}$$

That means that we should be able to use a mathematical similarity approach with six unknows to describe this transition. We have chosen a polynomial fit for the path in the x-y plane:

$$y = F(x) = a_0.x^6 + a_1.x^5 + a_2.x^4 + a_3.x^3.a_4.x^2 + a_5.x + a_6 \tag{6}$$

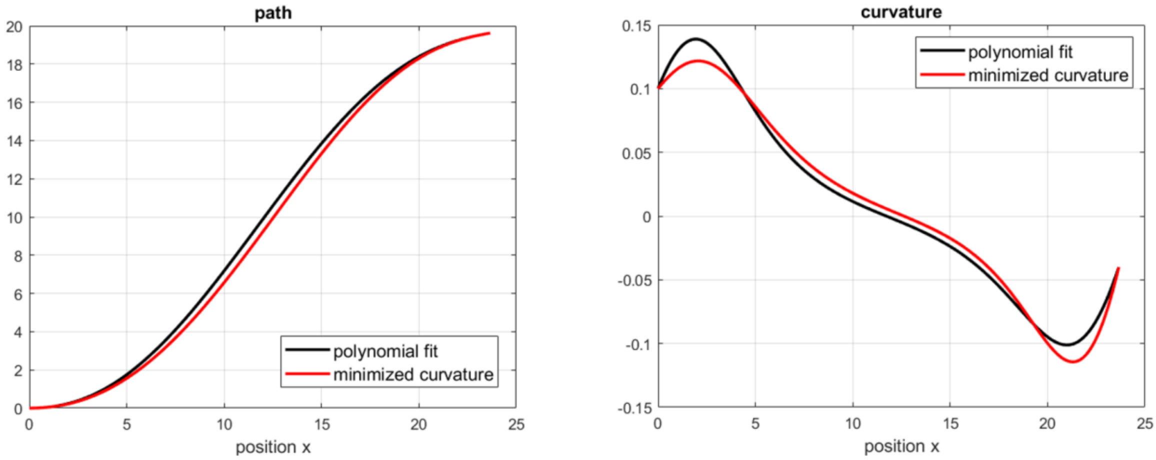

Figure 4.: Example of polynomial fitting forward transition path

This expression has one unknown (coefficient) more than required, which we use to minimize the curvature. Under steady state conditions, the front axle steering angle δ_s is related to the curvature κ (reciprocal path radius $1/R_s$ at tractor rear axle) according to $\delta_s = \arctan(\kappa. L_0)$ for tractor wheel base L_0. For sharp cornering, the steering angle is bounded and therefore also the curvature. That means that one needs to avoid high κ – values (small path radius). An example is shown in figure 4 with indication of the tuning of the additional coefficient to minimize κ. The peak value of 0.14 is reduced to 0.12. We have explored other similarity approaches and found no improvements with respect to the final transition path.
In figure 2, the start of the manoeuvre is related to zero slope and zero curvature ($f_0 = 0$, $k_0 = 0$). In this paper, we shall restrict to that. However, as indicated in figure 1, the vehicle

might start from a situation where slope and/or curvature are non-zero, as also shown in the above example. Likewise, the zero initial conditions are easily extended to more general situations.

5.2. The rearward part of the docking process, phase 3.

The reverse path for phase 3 is based on forward motion of the vehicle combination, starting with the rear end at the docking station, first in a straight line and followed by a motion by the tractor approaching a steady state circular path, with the semitrailer following. That is done by a ramp steer input, with a ramp time of 2 sec. As a consequence, the front axle will follow its circular path only after some (short) time, and the same holds for the tractor drive axle. The semitrailer axle will not follow a circular path directly but approach one after some time when steady state conditions are obtained, with smaller radius compared to the front axle. As indicated, the whole path is taken as the reverse path. Hence, for all three axles, there is a non-circular transition part before steady behaviour is reached. It is obvious that points P_2 and P_3 in figure 2 are positioned at the steady state circle of the rear tractor axle. The angle corresponding to point P_2 will be denoted by φ_b. As mentioned, more time is required for the trailer axle to reach the steady state circle. A situation is shown in figure 5. The ramp steer is initiated when the front axle is positioned at $y = 0$. Observe that the trailer axle needs in the order of 10 m beyond $y = 0$ to saturate to the steady state curve. If we demand an error less than 0.1 m (0.2 m), this is satisfied at y = 12.3 m (8.04 m). Throughout this paper, we shall take the radius R of the steady state circle for the tractor rear axle equal to $R = 22$ m.

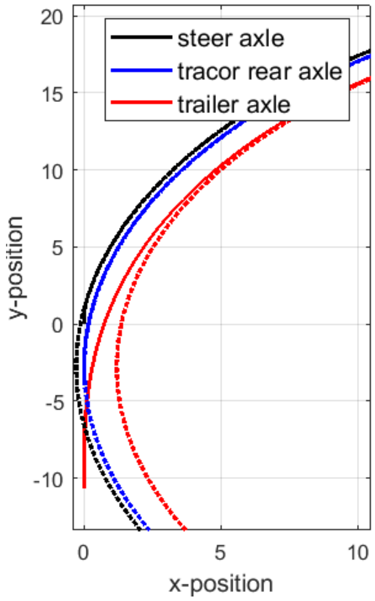

Figure 5.:Ramp steer response (dotted lines indicate the steady state circular behaviour)

5.3. Phase 2 of the forward transition part of the docking process

Now that we have indicated the three different reference reverse paths for the three axles, we use these to describe the second phase of the forward transition (reference) path. For a steady state situation, with the path radius R for the tractor rear axle, the corresponding radii for front axle and trailer axle are given by $R_f = \sqrt{R^2 + {L_1}^2}$ and $R_r = \sqrt{R^2 + f^2 - {L_2}^2}$, respectively. Denote these circles as C, C_f and C_r. When the rear axle 'hits' circle C at P_2 and follows it onward, the rear trailer axle will approach C_r. Phase 2 is defined as the part of the reference path until the deviation from circles C and C_r is within a predefined value, which has been chosen in this paper as 0.05 m. Let us start with $\varphi_b = 0.8 * \pi/2 = 1.257$ [rad] and $L_c = 2.5$ [m].

The transition paths for the three axles are shown in figure 6 also indicating the reverse paths. One observes that an acceptable match with reverse circles for tractor drive axle and trailer axle is obtained after 26,8 and 35.5 sec. respectively.

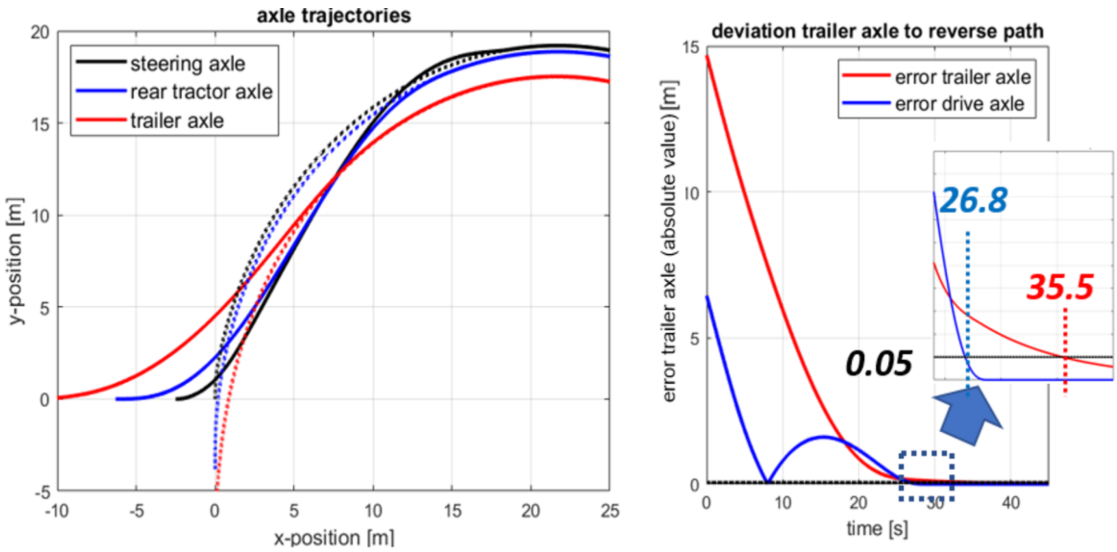

Figure 6.: Axle trajectories (reference curves) and absolute errors in approximation reverse path circles ($\varphi_b = 0.8 * \pi$[rad] and L_c=2.5 [m]). Dotted lines represent the intended reverse paths.

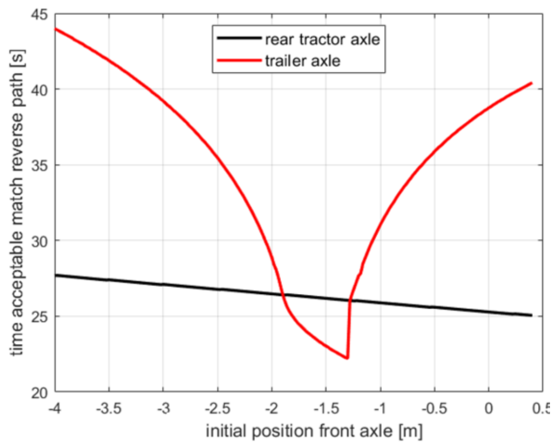

Figure 7.: Time to have error<0.05 [m] between reference paths for different axles and their respective reverse reference circular paths (φ_b=0.8 π/2 [rad])

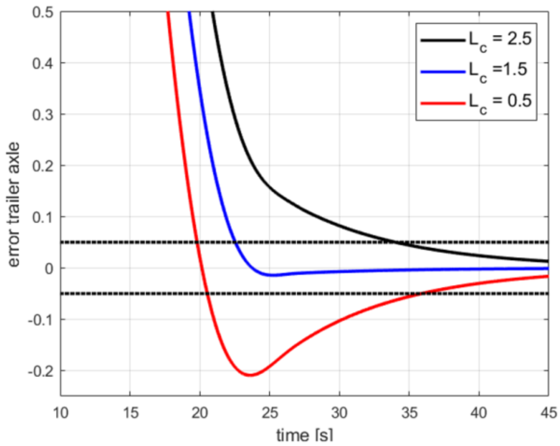

Figure 8.: Difference between trailer axle reference paths and circular reverse path

The front axle and rear tractor axle paths approach their reverse reference circles from inside these circles, whereas the rear trailer axle path is approaching its reference path from outside this circle. From the (absolute) error plot, one observes the overshoot of about 2 meters for the tractor axle, starting at roughly 8 sec., when the path intersects the circular path. Our next question is of course whether we could do this better, meaning reducing the time for acceptable error. We have first fixed the point P_2 and varied the initial position of the front axle, denoted by L_c. Results are shown in figure 7 in terms of the time for phase 1 and 2 together, i.e. to have an acceptable match with the reverse reference circles for both tractor drive axle and trailer axle.

There is apparently a range for L_c near 1.5 (initial front axle position -1.5) where the trailer reaches the acceptable error within a time between 22 and 24 sec and before this acceptable error is reached for the tractor drive axle at P_2. That means that we can skip phase 2. Starting further away leads to a larger time, which is obvious because one needs to travel more. But a change of L_c of 4.5 m only explains 4.5 sec (for a speed of $V = 1$ m/s), which is small compared to the differences in figure 7 for the trailer axle. Starting manoeuvring closer to the docking gate also leads to a larger time. We have examined the trailer axle reference path for three values for L_c, $L_c = 2.5, 1.5$ and 0.5. The errors between this path and the circular reference reverse path for the axle trailer for these values for L_c are shown in figure 8. For $L_c = 0.5$ the trailer axle path cuts the reference path for the reverse move, indicated with the red solid curve (change of sign), and then approaches this circular path (error → 0). The vehicle front axle starts very close to the reverse circles and therefore needs a large steering angle to correct it, with the consequence that the reverse path is only reached quite late by the rear trailer axle, around 35 sec. On the other hand, for $L_c = 2.5$, the vehicle starts too far away with again the consequence that the reverse path is reached late, but now without the trailer axle cutting this reference path. For the intermediate value $L_c = 1.5$, the trailer axle 'hits' its reference circular path earlier, at around 22 sec. The maximum steering angle, used to determine the forward reference path, varied between 28 and 35 degrees for $L_c = 2.5$ and 0.5, respectively.

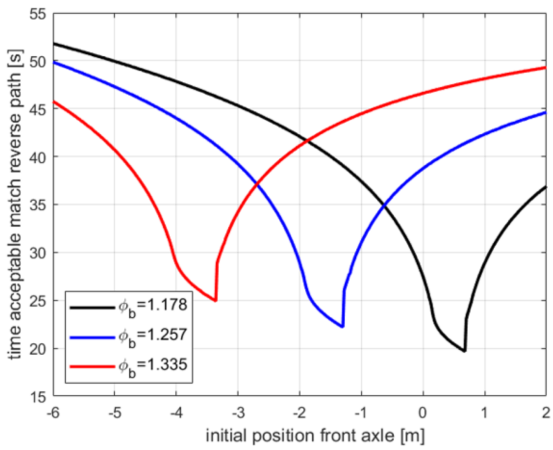

Figure 9.: Time to have error<0.05 [m] between different axle reference paths and their respective reverse reference circular paths for different Phase 1 end-points.

We have repeated the calculations for $2 * \varphi_b/\pi = 0.75$ and 0.85 ($\varphi_b = 1.178, 1.335$), determined again the time to have an error<0.05, and compared that to the previous case $2 * \varphi_b/\pi = 0.80$ ($\varphi_b = 1.178$). Results are shown in figure 9, which repeats figure 7, with the same plot added for the other two cases. The consequence of bringing point P_2 down along the reverse path is that the driver needs to start steering later, in order to have the shortest time (i.e. fastest docking manoeuvre) to match the reverse paths for rear tractor axle and trailer axle within the prescribed distance (0.05 m). Compared to the previous case, the optimal time has reduce from roughly 22.5 sec. to 20 sec. But there is a penalty for that, and that is an increased maximum steering angle to reach P_3. We have compared three cases, with the results listed in table 2. We have set the maximum. A tractor can reach larger steering angles. On the other hand, drivers like to keep their steering angle limited. When we set the maximum allowable steering angle at 35^o, the last situation in table 2 would not have been accepted.

φ_b [degrees]	L_c [m]	δ_{max} [degrees]
76.5	3.5	25.18
72.0	1.5	31.39
67.5	-0.5	39.75

Table 2.: Maximum steering angle for different positions for P_2 and initial positions of the front axle (forward reference path)

6. The vehicle docking manoeuvre.

The vehicle path is determined through a tracking procedure, with different characteristics in forward and rearward direction. In forward direction, we have taken a simple tracking model based on a single preview distance L_{pf} (measured from the front axle) and a steering gain K_{pf}. That means that the driver looks ahead and applies a steering angle δ being equal to steering gain times observed path deviation D_{pf}, see figure 10.
In the rearward direction, we have followed a similar approach as in [5], using inverse kinematics based on a virtual steering angle δ^*, and using a PI-controller for a preview distance L_{pr} also including the deviation in orientation. Using the notation as indicated in figure 10, this leads to:

$$\delta = K_{pf}.D_{pf}, \qquad \text{at preview length } L_{pf} \tag{7}$$

$$\delta^* = K_\theta.\left(\theta_2 - \theta_{pr}\right) + K_{pr}.D_{pr} + K_I.\int D_{pr}, \text{at preview length } L_{pr} \tag{8}$$

The parameters as used in the analysis are listed in table 3.

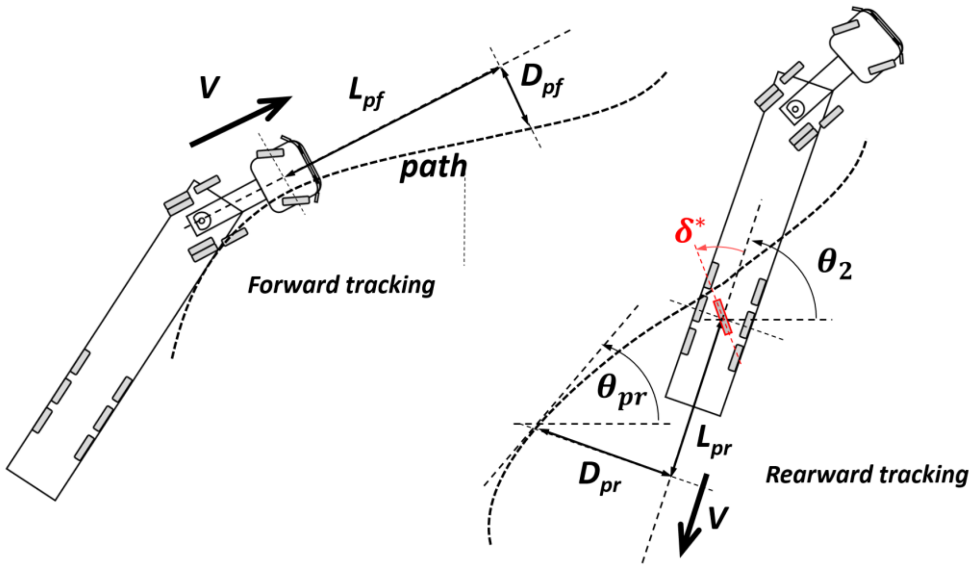

Figure 10.: Forward and rearward tracking layout

Parameter	*Value*
L_{pf}	1.6 [m]
K_{pf}	0.51 [rad/m]
L_{pr}	0.4 [m]
K_{θ}	18
K_{pr}	1.0
K_I	0.003

Table 3.: Tracking parameters

Kural used the PI driver model for both forward and rearward tracking, in both cases tracking the path from the rear axle. He used two preview points instead of one, according to Salvucci and Gray [11]. We shall show that one preview point is sufficient to have a very accurate docking manoeuvre. Results for forward tracking are shown in figure 11 in terms of the error with the reference paths from the previous chapter for the different axles. We have taken the second case in table 2. The integration has been continued beyond the point where phase 3 starts, to show the further convergence to the reverse circles (indicated as dashed lines).
We see that the path errors for the rear tractor axle and the trailer axle finally end up to be smaller than 0.05 m. Maximum errors are found for the front axle (around 0.2 m) and rear

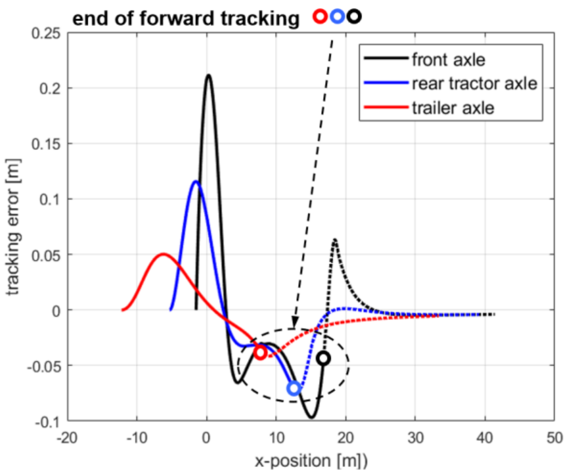

Figure 11.: Path for the three axles for phases 1 and 2 (forward tracking)

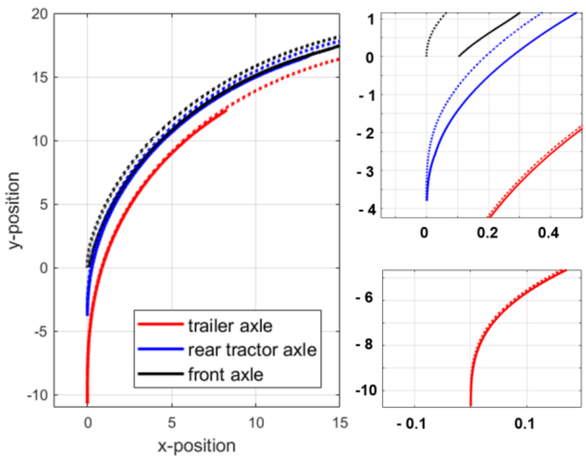

figure 12.: Trajectories vehicle axles (rearward tracking)

tractor axle (0.12 m) at the location of the docking gate. This is also close to the location where the steering angle is maximal, i.e. where the path curvature is largest. The vehicle axle trajectories are therefore close to the behaviour, shown in figure 6.
The accuracy during rearward driving to the docking gate is remarkable, see figure 12. The left plot shows the overall behaviour while reversing, whereas the two smaller plots are enlargements near the final positions of the three axles. The path error for the trailer axle converges to zero and is very small all along the reverse manoeuvre. The same holds for the rear tractor axle. The front axle path ends with en error of about 0.1 m.

5. Conclusions and discussion.

The research, presented in the paper, discusses docking performance of a tractor-semitrailer combination for a certain docking scenario, and derives the conditions for which the total docking time is small, i.e. when the docking gate is reached fast. The objectives of this research as listed in chapter 2 are fulfilled. The errors in path tracking are extremely small, specifically during the last phase of the docking manoeuvre using a single preview point PI control, with the trailer axle reverse tracking error converging to zero. In practice, it may be difficult to reach such an accuracy at a docking gate, but this paper shows the potential of the controller set-up to dock successfully in an automated way or through an adequate driver assistance system. Secondly, the total docking procedure can be considered to be optimal in the sense that the total procedure takes very limited time. This is all about the onset of the steering for positioning the vehicle combination to start the reverse parking manoeuvre towards the docking gate. Doing that right, the rear trailer axle is more or less directly pulled to the correct position for accurate rearward follow-up path tracking.
In addition, we make the following remarks:

- The results were discussed with truck drivers, and our algorithm appears to predict the actual driver performance rather well in qualitative sense.
- We have discussed Dubin's approach with a reference path built up from circular arc and straight segments. Instead of Dubin's approach, we followed the approach of deriving a transition reference path between the original straight path motion to an almost circular path (the reference path for the reverse of the manoeuvre). This approach can easily be extended to more complex transitions. Consider figure 1, where the shunt vehicle docking is proceeded by a circular forward motion instead of a straight path. More general, one could start with circular arcs and straight segments, to be linked by transitions as derived for phase 1, and discussed in section 5.1. This leads to a straightforward extension of Dubin's approach. '
- The algorithm can be used for automated vehicle control on a DC in practice, either in an automated fashion or through a driver assistance system based on this algorithm. The HAN-University is involved in various projects in this field. An example is VISTA [13] where the driver support approach is examined. A second example is CATALYST where we aim to establish automated vehicle behaviour, using path planning based on the A^* algorithm and lattice based motion planning technique. The input to the path planner is taken as a motion primitive library, determined off-line and referring to motion segments from one vehicle state to another, respecting the vehicle's kinematic constraints and a cost (minimized path length) function. It accounts for static and slowly moving obstacles. A more extensive treatment can be found in [3].
- Investigation of vehicle behaviour at a DC reveals that speed is not constant, and may be larger than 1 m/s. However, during the reverse parking manoeuvre, the vehicle drops to low values again. This leads to the discussion whether a dynamic model should be applied instead of a kinematic model. Together with Eindhoven University of Technology, it is intended to consider this more rigorously.

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