

Base Line Performance of the Low Cost LORADD SP Receiver

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Introduction

Global Navigation Satellite Systems (GNSS) currently dominate the Positioning, Navigation and Timing (PNT) community and there are no signs that this will change in the near future. But for many years the weaknesses of GNSS have been well known and besides research into GNSS a lot of research into back up systems like Inertial Navigation Systems (INS) and eLoran has been done.

For military operations high availability of PNT data is of utmost importance. We estimate that the probability that GNSS is not available for a long period of time and in conjunction with that the risk that a military operation fails due to a lack of continuity of service is very small. In view of this an expensive back up system is not justified in our opinion. Therefore we began research into a low cost eLoran receiver. To this end we purchased the LORADD SP eLoran receiver made by the Dutch firm Reelektronika. The main data produced by this receiver are position, time and heading.

Besides low cost a back up system should at least meet the requirements for positioning and heading as stated in Tables 1 and 2. The requirements have been taken from several civilian documents since the Netherlands Armed Forces currently do not have an official policy on PNT¹.

¹The most recent version of a PNT policy for the Royal Netherlands Navy dates back to the mid nineties of the twentieth century. Whether the Royal Dutch Army and Airforce have a official PNT policy is not known.

Table 1: Position accuracy requirements according to different sources.

Source	Accuracy (95%) [m]
FRP2008 Table 4-3	8-20
ERNP Table 69	10
IMO Resolution A953(23)	10

Table 2: Heading accuracy requirements according to IMO Resolution MSC116(73).

Error type	Max. error [°]	Rate of Turn [°/s]
Static	1.0	0
Dynamic	1.5	0
Follow up	0.5	0-10

In this article the positioning and heading performance of the LORADD SP receiver will be investigated and tested against the requirements. Because it is a preliminary research into eLoran at the NLDA the research is also meant to give ‘hands on’ experience in this field and to generate research topics.

Loran-C and eLoran

Loran-C is a positioning system operating at 100 kHz that measures time differences (TDs’) between carefully shaped pulses that are transmitted by synchronised transmitters; these TDs’ can be transformed into the position of the receiver antenna. Enhanced Loran (eLoran) is an improvement of Loran-C that measures the time of arrival (TOA) of a pulse and using the time of departure (TOD) of this pulse calculates the travel time τ the pulse needed to propagate from the transmitter to the receiver antenna. τ can be transformed into the distance d_{TR} between transmitter and receiver antenna by multiplying with the speed of light c :

$$\tau = TOA - TOD \Rightarrow d_{TR} = c \cdot \tau$$

We can write d_{TR} , which is the distance along the geodetic line, as a function of the known latitude and longitude of the transmitter, the unknown latitude and longitude of the receiver antenna and the unknown clock error of the receiver yielding a relation -the observation equation- with one measurement (TOA) and three unknowns which can be solved when measurements of at least three transmitters are available.

When used in combination with an H-field antenna the receiver is capa-

ble of measuring the direction from the receiver to the transmitter relative to the reference direction of the antenna. Based on the known position the true bearing towards the transmitter can be calculated. At any epoch t the relation between heading $H_{loran}(t)$, calculated bearing $B_{calc}(t)$ and measured relative bearing $RB_{obs}(t)$ reads (see Figure 1):

$$H_{loran}(t) = B_{calc}(t) - RB_{obs}(t) \quad (1)$$

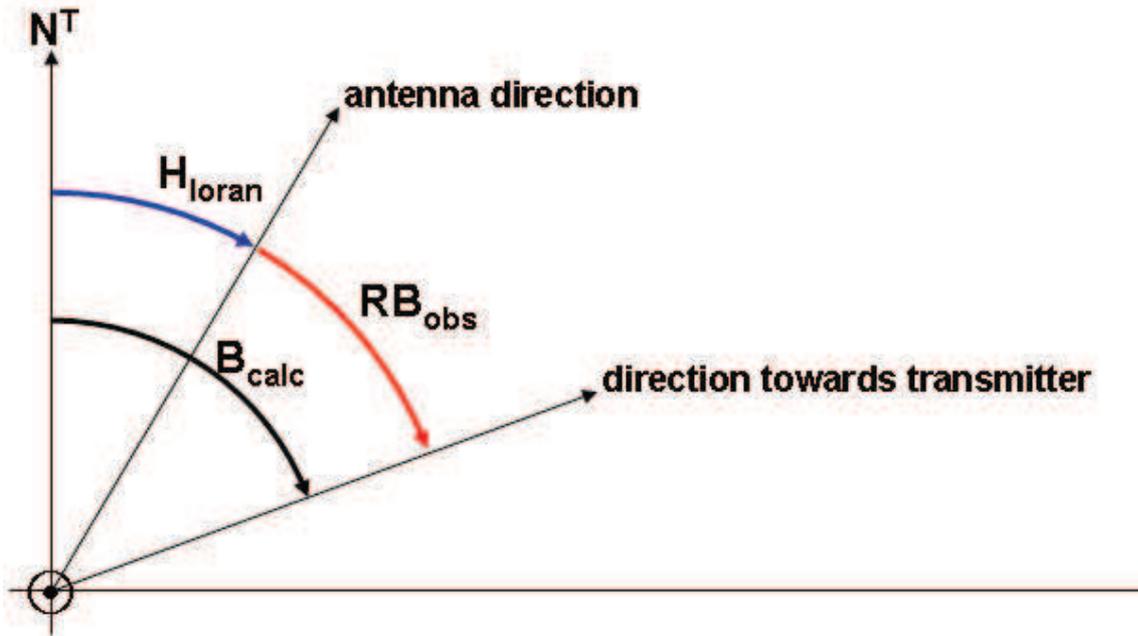


Figure 1: Relation between heading, observed relative bearing RB_{obs} and bearing B_{calc} relative to true north N^T .

According to Table B-3 of [5] the positioning accuracy of Loran is 460 m (95%); the heading accuracy is not stated. The accuracy of eLoran is not (yet) stated. For a comprehensive description of (e)Loran see [1].

The LORADD SP receiver

For our research we used the LORADD SP Integrated GPS/eLoran receiver of the Dutch firm Reelektronika. It can be used either with an E-field antenna or with an H-field antenna. It is a so-called all in view receiver which means that it tracks the signals of all transmitters it can receive.

Although it can track many signals only the ones with a certain (unknown to us) signal-to-noise ratio are used for the calculation of the position. For positioning at Den Helder Roads the receiver only used the signals of transmitters in Sylt (Germany), Lessay (France) and Anthorn (England). The receiver integrates measurements over a period of 5 seconds before processing yielding one eLoran position every 5 seconds.

Besides eLoran measurements, the receiver is capable of measuring GPS ranges. Using Eurofix range and range rate corrections these measurements produce DGPS positions. (D)GPS and eLoran can be used stand alone or integrated. When both (D)GPS and eLoran are available (D)GPS can be used to calibrate eLoran giving an improved performance of eLoran when (D)GPS is temporarily unavailable due to for instance interference or jamming.

When used in combination with an H-field antenna the receiver can produce heading. It is not clear whether the receiver uses the strongest signal for the heading, or the signal from the nearest transmitter or depending on their signal-to-noise ratio a combination of measurements from all transmitters in view. Heading measurements are integrated over one second intervals before processing.

For a comprehensive description of the LORADD SP receiver see [2].

Preliminary remarks and assumptions

Due to the fact that it is unknown which and how many measurements are used for the heading, the performance of the heading output might change when the relative geometry of the transmitters change i.e. when the measurements are made in another area. Therefore all measurements have been made at Den Helder Roads and vicinity only (see Figure 2). As a consequence the results will only be valid for this area.

The heading output of the receiver is calculated using Formula 1 for which the true bearing $B_{calc}(t)$ to the transmitter has to be calculated. The distance to the nearest transmitter in combination with the expected accuracy of Loran (460 m) are such that an error in $B_{calc}(t)$ due to a random shift in position may be ignored. The measured headings have only been corrected for relative bearing dependent errors which are inherent to the

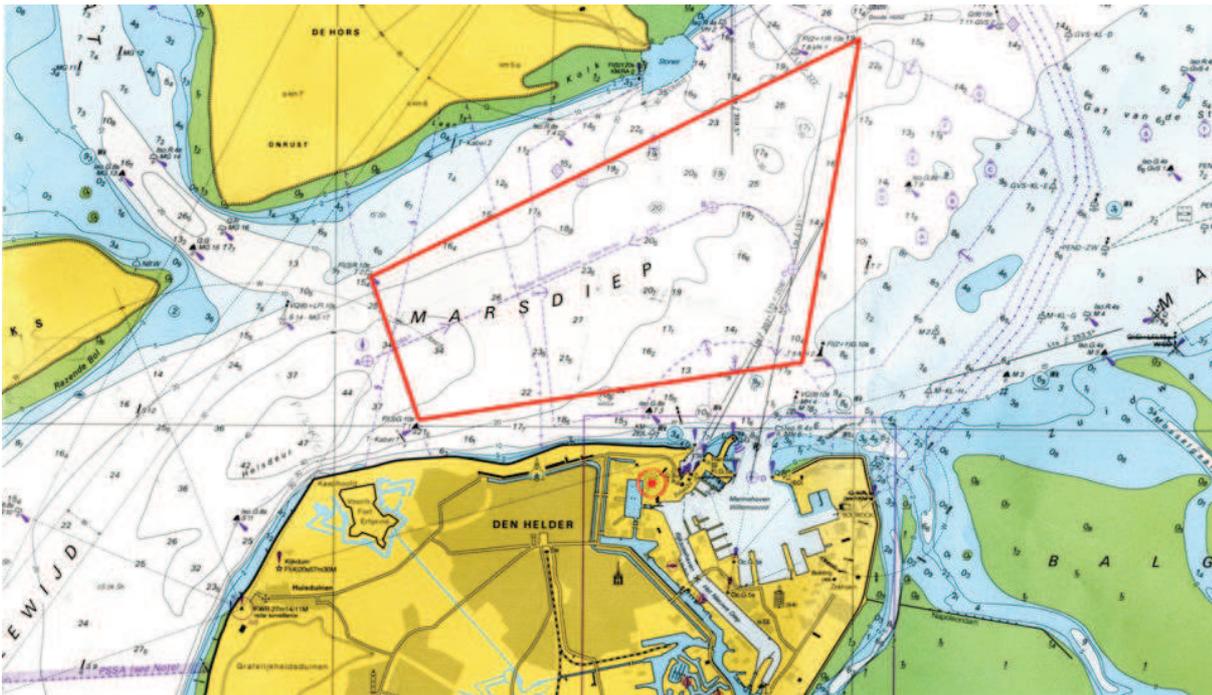


Figure 2: Copy of a portion of Netherlands Nautical Chart 1546, ‘North Sea Netherlands, Zeegat van Texel and Den Helder Roads’. The antenna position for the static measurements is marked by the red circle. The red lines indicate the track sailed for the dynamic measurements.

H-field antenna.

To assess the accuracy of the position of Loran we started the research using measurements without any corrections except the corrections known as Primary Factor (PF) and Secondary Factor (SF). This yielded a base line accuracy to be used as a reference to assess the effect of future improvements (see the next article).

All static measurements have been done on the roof of the Klooster, one of the buildings of the Netherlands Defence Academy (NLDA) in Den Helder. All dynamic measurements have been done on a small harbour tug (see Figures 3 and 4 respectively). The fieldwork has been done by Jelle Woudstra, Hessel de Groot and John den Ouden, students at the NLDA.

Position measurements

Static positions

To assess the accuracy of the position in the static case we measured DGPS ranges over a period of 48 hours and calculated the position of the com-



Figure 3: Left: Aerial view of the Klooster, one of the buildings of the NLDA at Den Helder. Right: Close up of the antenna location on top of the Klooster.

bined GPS/eLoran antenna on the roof of the Klooster. This position ($52^{\circ}57'43.248''\text{N}$, $004^{\circ}46'26.668''\text{E}$) has been used as a reference position throughout.

To gain a first impression of the accuracy eLoran signals of Sylt, Lessay and Anthorn were observed and logged at 5 second intervals from 13 to 20 february 2009. In Figure 5 the positions are plotted relative to the reference position. The mean eLoran position is in (205.8, -159.7) which is 260 m in the direction 128° relative to the reference position. The covariance matrix



Figure 4: Harbour tug Wierbalg used for dynamic eLoran measurements.

of the observed positions in $[m^2]$ reads:

$$\begin{pmatrix} 20.4 & -18.3 \\ -18.3 & 24.3 \end{pmatrix}$$

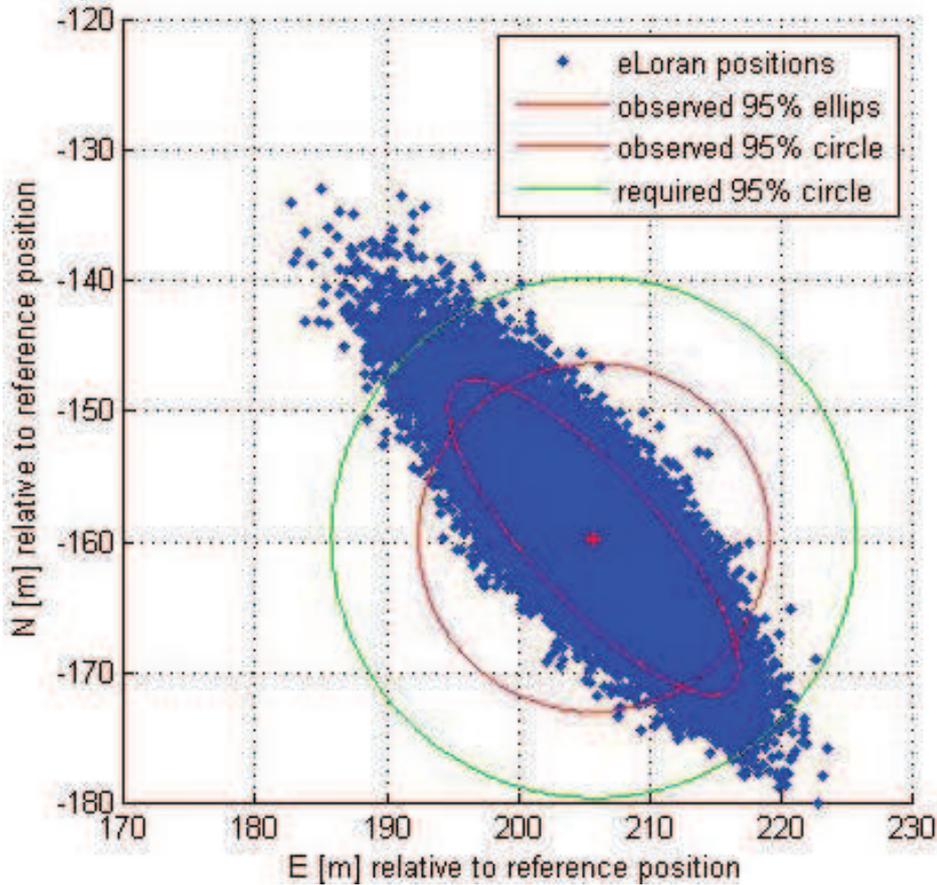


Figure 5: Scatter plot of eLoran positions observed at the Klooster between 13 and 20 february 2009.

Although there is a large bias, the spread of the eLoran positions around the mean value is rather limited: the 95% ellipse has a semi major axis in the direction 138° - 318° with a length of 15.6 m; the semi minor axis has a length of 4.9 m. The radius of the 95% circle equals 13.4 m. This indicates, as expected from literature, that the absolute accuracy of Loran is poor while the precision is good.

The direction of the semi major axis nearly coincides with the direction towards Anthorn (295°). The antenna position is very close to the baseline between Sylt and Lessay. Based on that we may conclude that

the orientation of the ellips and the bias are mainly caused by the fact that there is only one transmitter (Anthorn) producing one position line in the NE-SW direction while there are four position lines in the NW-SE direction because Sylt and Lessay are both dual rated.

As can be seen in Figure 5 the center of the ellipse which is the mean position over the observation period is not in the center of the scatter plot, but slightly to the south east. This might indicate that the mean position changes over time. Therefore we calculated the mean positions and mean R95 over periods of one hour. This indicated a change in position error between 253 and 270 m and a change in R95 between 6 and 16 m (see Figure 6. Note however that the position error minus 250 m is plotted.). Since there seems to be a relation between these changes and day and night we also calculated the mean position error and R95 during the day (07.00 -17.00 UTC) and night (17.00 - 07.00 UTC). In Figure 7 these values are indicated as asterisks.

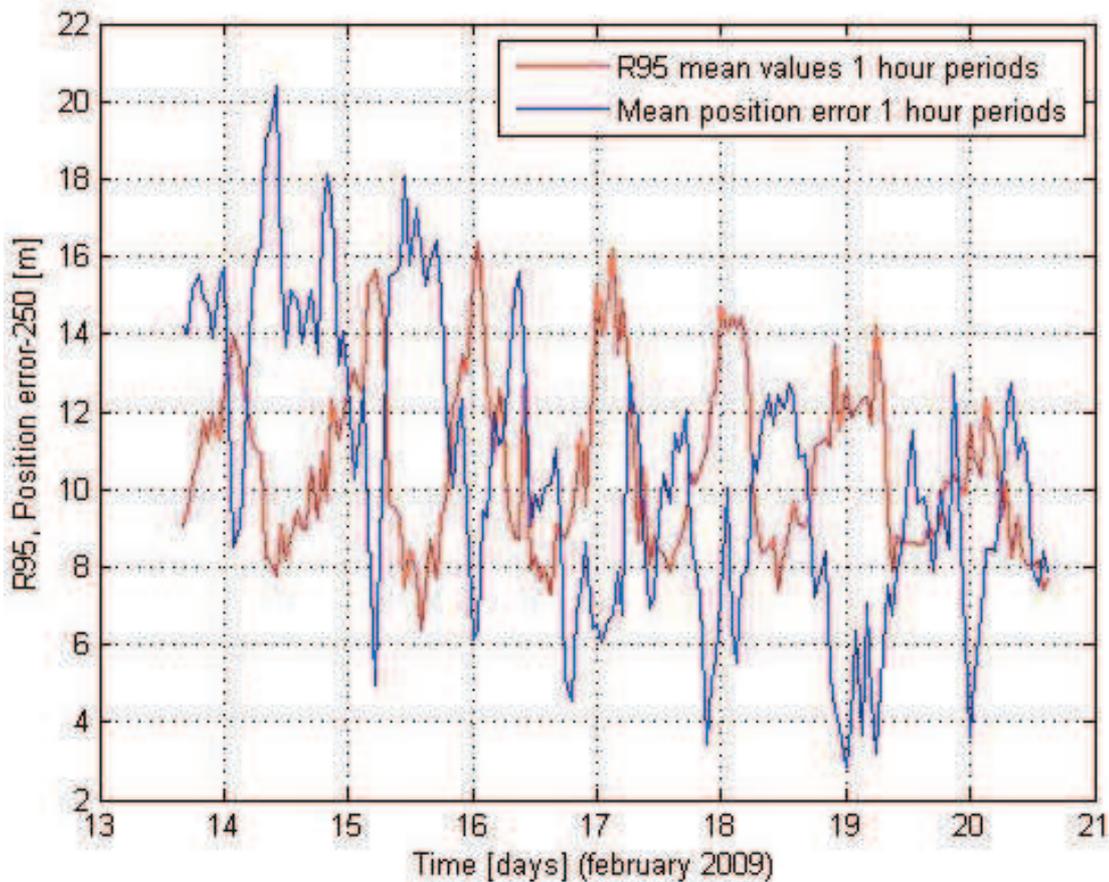


Figure 6: Position errors and values for R95 averaged over one hour periods.

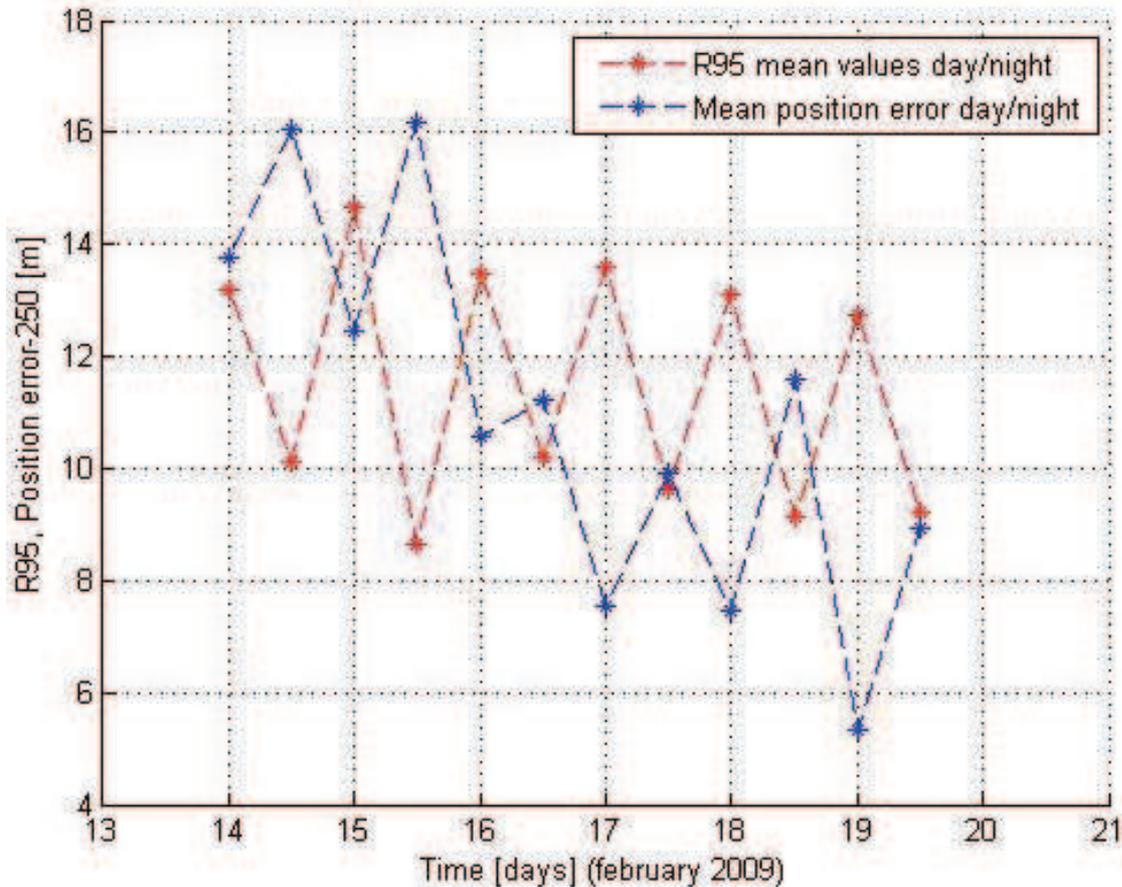


Figure 7: Position errors and values for R95 averaged during the day and night.

At Den Helder Roads the base line accuracy of eLoran positions in the static case is well within the 460 m as stated in Table B-3 of [5] but does not fulfill the requirements of Table 1. However, since the radius of the observed 95% circle is smaller than the radius of the required 95% circle (in its most relaxed version) the precision does meet the requirements.

From literature it is known that H-field antennae produce relative bearing dependent errors in position [1,3]. In order to assess the influence of the antenna direction on the bias and 95%-ellipse we measured positions in 8 different directions during 15 minutes (180 positions) in each direction. These measurements revealed that the distance between the reference position and the eLoran position varied in the direction of the semi major axis with plus or minus 40 m relative to a mean eLoran position as a function of the antenna direction (see Figure 8). An influence of antenna direction on the precision (size and orientation of the ellipse) was not observed.

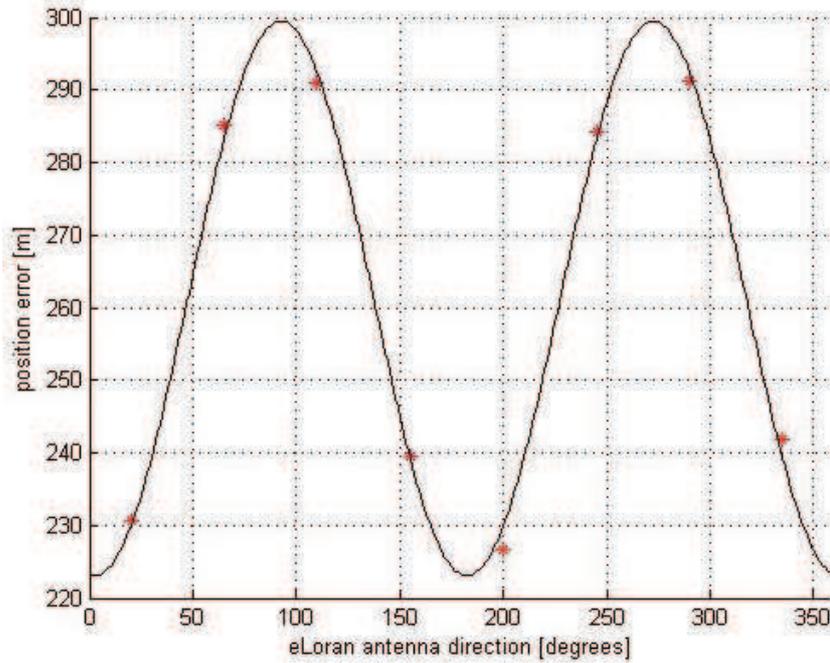


Figure 8: Position error as a function of antenna direction.

Dynamic positions

To get an impression of the degradation of the eLoran position in dynamic circumstances we compared calibrated eLoran positions with positions of an independent Trimble agGPS332 DGPS receiver, which served as the reference (ground truth). The positions were logged sailing the same circuit in Den Helder Roads twice.

Figure 9 gives the scatter plot of these positions. Since eLoran was continually calibrated with the LORADD DGPS we expected the accuracy to be in the order of the accuracy of DGPS (5 m (95%)). But although the 260 m bias has been removed by the receiver the precision has decreased in comparison to the static case. The radius of the 95% circle has increased to 28 m. When we look at the scatter plot of Figure 9 the increase of the R95 is probably caused by the relative bearing dependent error because several clusters of positions are apparent.

The accuracy of eLoran positions that are continually calibrated does not meet the requirements of Table 1.

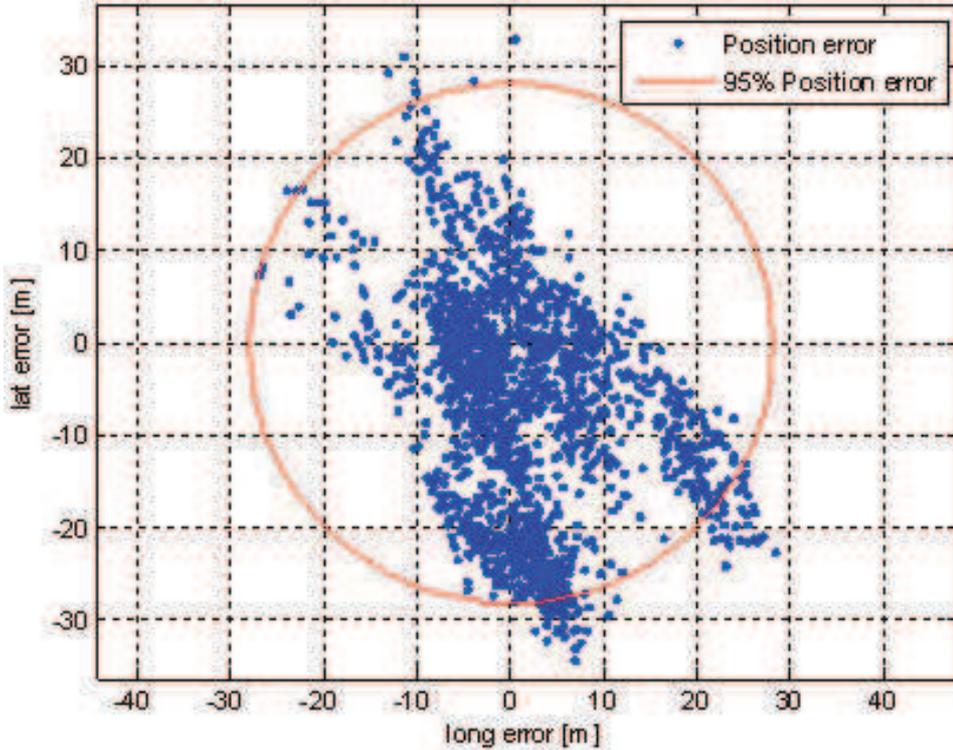


Figure 9: Observed errors between DGPS and calibrated eLoran sailing a circuit in Den Helder roads. Due to the relative bearing dependent error position errors are in clusters. Note the increase of the R95 compared to Figure 5.

Heading measurements

As well as the range measurements, the heading measurements also suffer from relative bearing dependent errors. To correct for these errors we constructed a correction function which gives the correction to be applied to the observed eLoran heading as a function of the relative bearing towards Sylt. Since the main source of these errors (re-radiation by objects in the vicinity and/or by the own vessel) was different on the roof of the Klooster and on board of the tug we constructed a function for the static measurements and one for the dynamic measurements.

For the construction of the correction function we compared $H_{loran}(t)$ with $H_{phins}(t)$, the heading yielded by PHINS for headings between 000° and 360° with steps of 30° . The correction $\Delta H(t)$ in $[\circ]$ at epoch t as a function of the heading reads (see Figure 10):

$$\Delta H(t) = H_{phins}(t) - H_{loran}(t) \quad (2)$$

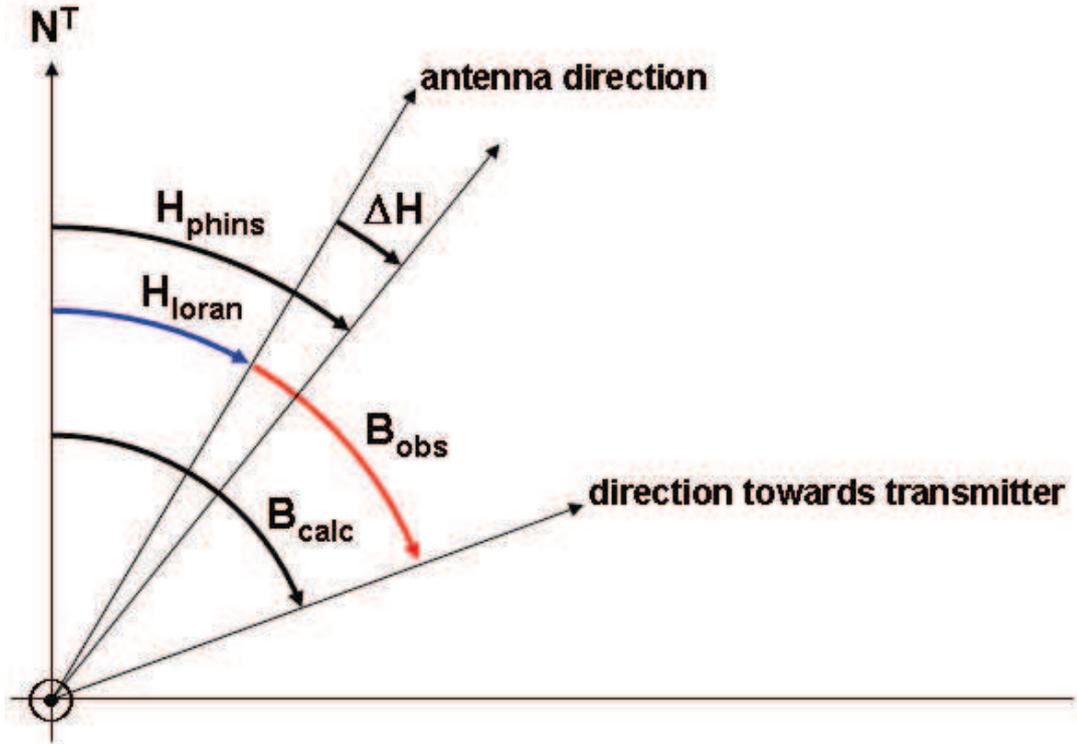


Figure 10: Relation between heading correction ΔH , observed eLoran heading H_{loran} and observed reference heading H_{phins} . PHINS is an accurate inertial navigation system that was used throughout the research to produce an accurate reference heading.

$\Delta H(t)$ consists of a systematic part which is mainly due to a difference in alignment between the eLoran antenna and the PHINS and a relative bearing dependent part.

In order to construct a function that is independent of time and in order to minimize the influence of random errors the measurements have been repeated several times. From literature [1,4] it is known that the correction function is a cosine with two periods in 360° and may well be written as a Fourier series.

Static headings

Using a least squares adjustment the next correction function has been derived for the static case (see Figure 11):

$$\begin{aligned} \Delta H(B) = & 1.63 + 0.53 \sin(B) + 0.24 \cos(B) \\ & + 0.77 \sin(2B) + 1.67 \cos(2B) \\ & - 0.01 \sin(3B) - 0.15 \cos(3B) \end{aligned}$$

In this function B is the relative bearing towards Sylt, which is the transmitter that is nearest and normally has the highest signal-to-noise ratio in the area Den Helder Roads. Terms of an order higher than three turned out to be negligible. The adjustment was based on 34 sets of measurements in four days in 12 directions. Each set consisted of about 600 measurements of $\Delta H(t)$.

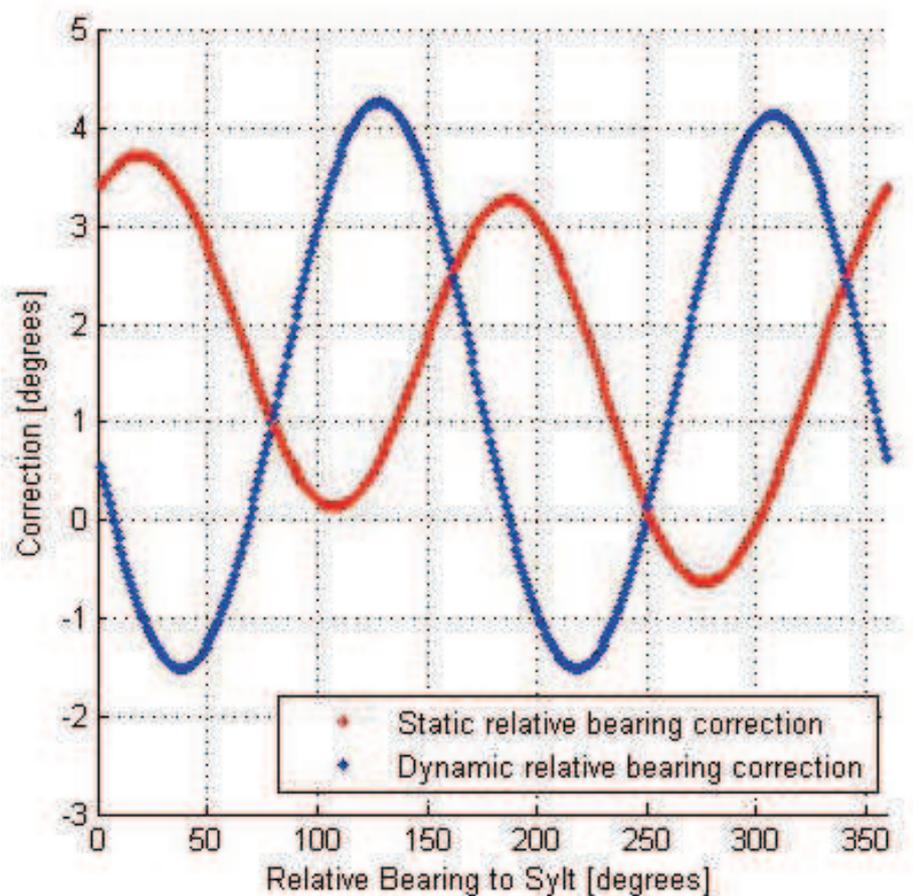


Figure 11: Correction to be applied to eLoran heading readings as a function of the relative bearing towards Sylt.

The mean value and standard deviation of each dataset was calculated after correction. This yielded mean heading corrections between -0.68° and $+0.59^\circ$ and standard deviations between 0.16° and 0.39° (see Figure 12). The overall mean value turned out to be 0.01° with a standard deviation of 0.46° [9].

In IMO Resolution MSC116(73) the static error is defined as the “error

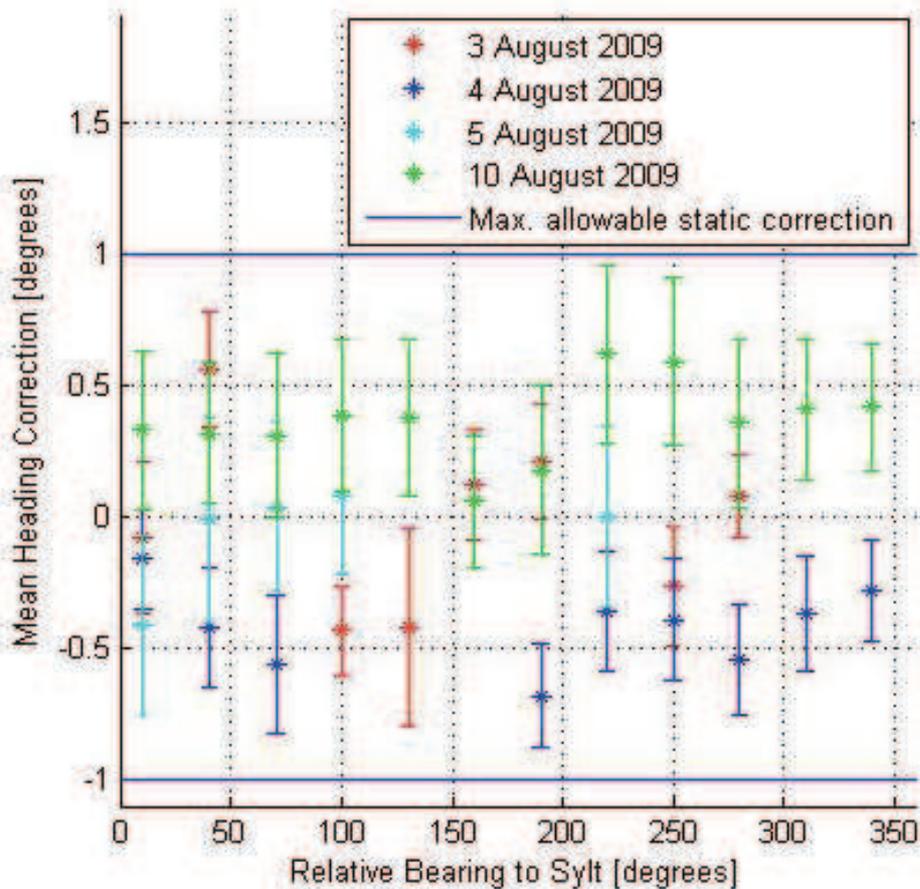


Figure 12: Mean heading corrections and standard deviations as a function of the relative bearing to Sylt.

which is caused by any reason and which stays unchanged in value during the operation of the system”. According to the requirements the maximum static error is plus or minus one degree. When we assume that a static error is the same as a systematic error, which is a plausible assumption in view of the definition, the LORADD eLoran compass fulfills this requirement the mean value of all individual sets and the overall mean value being smaller than one degree.

However because the definition is not quite clear the one degree maximum requirement could also mean that the probability that the static error exceeds one degree is zero². Assuming a normal distribution we then could argue that the one degree is equal to three times the standard deviation. In that case the compass might not fulfill the requirement since the standard

²Of course a probability will never be zero, but for a normally distributed stochastic variable it is known that the probability that the variable exceeds a value three times its standard deviation is only 0.3%. For practical reasons this probability can therefore be taken as zero.

deviation of a number of sets and the overall standard deviation exceed 0.33° .

Dynamic headings on a steady course

To derive the coefficients for the correction function to be used on the tug a circuit on the Marsdiep has been sailed three times in two days yielding some 20,000 measurements in all directions although not evenly distributed. The correction function for the dynamic measurements reads (see Figure 11):

$$\begin{aligned}\Delta H(B) = & 1.35 + 0.05 \sin(B) - 0.04 \cos(B) \\ & - 2.78 \sin(2B) - 0.68 \cos(2B)\end{aligned}$$

Again B is the relative bearing towards Sylt. In this case terms of order three and higher could be neglected. Since the re-radiation is only due to the tug itself the function resembles more the theoretical one with dominant terms of order two.

The mean value and standard deviation of the three corrected sets varied between -0.02° and 0.13° and 0.29° and 0.40° respectively.

IMO Resolution MSC116(73) defines the dynamic error as an “error which is caused by dynamic influences such as vibration, roll, pitch or linear acceleration.”.

For the requirement of dynamic errors the resolution states: “The dynamic error amplitude should be less than ± 1.5 degree. The dynamic error frequency should be less than 0.033 Hz equivalent to a period not shorter than 30 s if the amplitude of the dynamic error exceeds ± 0.5 degrees.”. The resolution does not state what the frequency is allowed to be in case the amplitude of the dynamic error is smaller than 0.5° . Since amplitudes are normally always positive the “ \pm ” might indicate that ‘about’ 0.5° is meant.

Also in this case it is not clear whether the 1.5° requirement is the systematic part of the dynamic error (on which a time varying or random part is superimposed) or a (nearly) three sigma value. No matter which one is meant, in both cases the compass fulfills the requirement.

Dynamic headings on a changing course

According to IMO Resolution MSC116(73) the follow up error is an “error which is caused by the delay between the existence of a value to be sensed and the availability of the corresponding signal or data stream at the output of the system. This error is e.g. the difference between the real heading of turning vessel and the available information at the output of the system.”.

Denoting the delay by k [s], the next relation between rate of turn $R(t)$ [$^{\circ}/s$] and follow up correction $\Delta H(t)$ is to be expected:

$$\Delta H(t) = kR(t)$$

In order to assess the follow up correction of the eLoran compass circle tests have been done with different rates of turn turning port as well as starboard. During each test the heading measured with PHINS at a certain epoch and the eLoran heading at that same time were measured and subtracted (see Formula 2). For each circle test the mean value of the follow up correction and its standard deviation were calculated. The results are plotted in Figure 13. As can be seen in Figure 13 the relation between the rate of turn and the follow up correction tends to be linear as expected. Using the calculated corrections the value for k was estimated to be 0.2. But it is very remarkable and unexpected that there is a bias of two degrees between the theoretical expected relation and the relation derived from the measurements. This bias is probably due to a misalignment of the antenna and/or PHINS.

Given the calculated corrections, the eLoran compass does not fulfill the requirements at all. However assuming the value for k is correct and the bias is due to errors in the measurements or in the measurement set up the compass might fulfill the requirements for rates of turn between zero and 2.5 degrees per second. For higher rates of turn the compass does not meet the requirements.

Discussion

The base line accuracy of eLoran does not meet the requirements. The scatterplots of the static as well as the dynamic positions show ellipses pointing towards Anthorn. This indicates that the ellipticity is mainly caused by random errors in the measurements of this transmitter. The

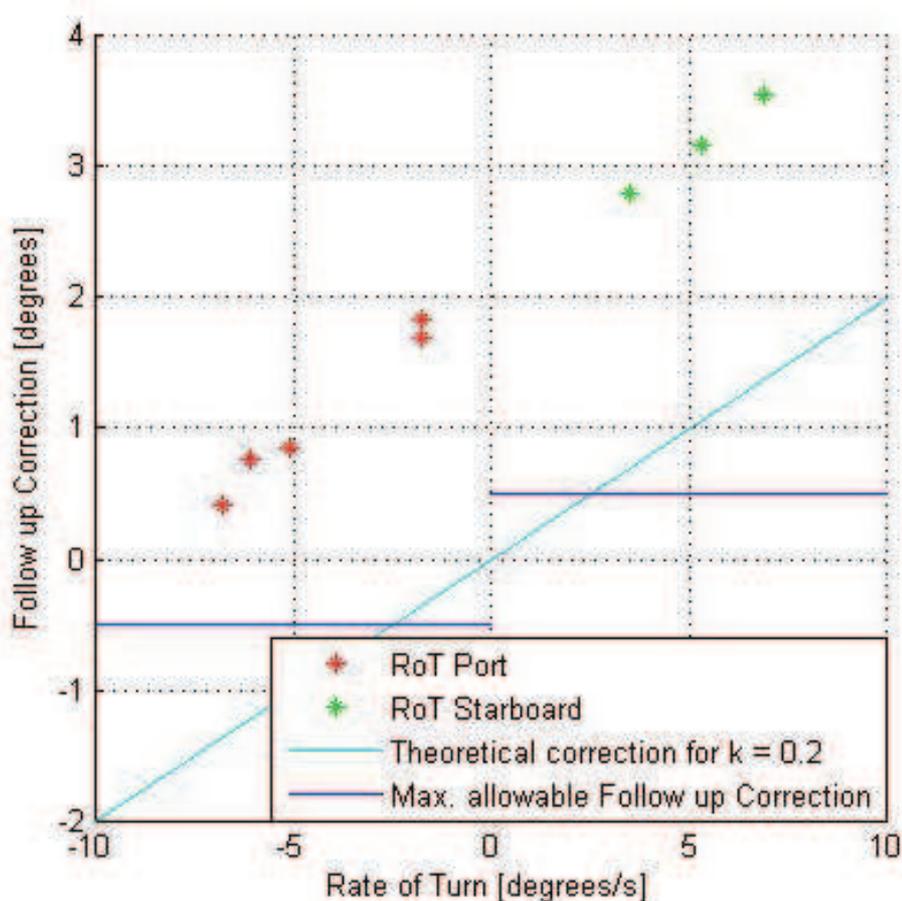


Figure 13: Follow up correction to be applied to the eLoran heading as a function of the rate of turn.

size of the ellipses of the dynamic positions was larger than that of the static positions. The reason for this is not clear from the measurements so far.

In the static case the mean positions show a bias that is directed towards/from Anthorn and changes along that direction as a function of antenna direction and as a function of the time of the day. Due to the fact that in the area Den Helder Roads the transmitters at Sylt and Lessay are nearly opposite each other all common errors in the signals of these transmitters will cancel yielding a better position accuracy in the direction Sylt-Lessay. Also in the dynamic case the mean positions show a bias, but the direction of the position shift is not towards/from Anthorn. When these biases turn out to be reasonably constant over a longer period of time we might transform them into (relative bearing and position dependent) corrections to remove them and improve the accuracy. Therefore a (plat-

form dependent) correction function for positions should be constructed analogous to the correction function for the headings.

Depending on the interpretation of the definitions in [6] the LORADD eLoran compass does or does not meet the requirements in the static and the dynamic case. According to the LORADD SP fact sheet the compass performs “better than one degree under normal conditions”. Although no definition is given of the term normal conditions the measurement results tend to confirm this statement. One should keep in mind that these results only hold when no re-radiation is present.

The performance of the LORADD eLoran heading when the platform has a rate of turn cannot yet be assessed. The trend in the graph of Figure 13 is in accordance with the theoretically expected relation, but the values of the corrections are not.

It is questionable whether the compass will be able to meet the requirements for the follow up error at all. Suppose a rate of turn of ten degrees per second to starboard and suppose the PHINS and eLoran heading at the beginning of the one second integration interval both are 000° . At the end of the integration interval the PHINS heading will then be 010° , but the integrated eLoran heading will be 005° . This yields a follow up error of at least 5° (one order worse than the requirement of 0.5°). Therefore the compass will probably not meet the requirements unless a correction is applied somewhere in the processing of the measurements.

Although H-field antennas have many advantages over E-field antennas [1] the main disadvantage is the fact that relative bearing dependent errors show up in the position as well as in the heading measurements. Therefore the use of an E-field antenna might be preferred especially in the case of maritime use of eLoran.

Conclusions

In this article the positioning and heading performance of the LORADD SP receiver have been investigated and tested against the requirements. The positioning and heading accuracy are summarized in Table 3.

Tested against the requirements the main conclusion is that as far as positions are concerned eLoran in its baseline configuration cannot serve as

Table 3: Summary of position and heading accuracy.

	Mean value	Accuracy (95%)
Position	260 m	12.4 m
Heading Static	0°	0.9°
Dynamic	0°	0.8°
Follow up	as yet unknown	as yet unknown

a back up for GNSS. However the measurement results are very promising in that way that the performance might be improved by removing the bias in the position and by applying a relative bearing dependent position correction.

The LORADD receiver can serve as a back up for a compass when no re-radiation is present.

Besides testing the performance of the receiver another research goal was to gain ‘hands on’ experience in the field of eLoran and to generate research topics. Although the LORADD receiver does not meet the requirements the knowledge of the possibilities of the receiver has increased. Besides this the results give rise to a number of research topics to improve performance.

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