From the Lab to the Sea, Acoustic Sensing in Uncertain Environments

Vincent van Leijen

***1990.** Somewhere above the Atlantic Ocean a lone maritime patrol aircraft is on an ASW mission. Directed by SOSUS intelligence the crew is ordered to monitor a designated area with a field of sonobuoys. The first buoy to hit the water is an expendable bathythermograph (XBT). The device samples the temperature profile in the water column and the operator quickly derives a sound speed profile. After the propagation conditions of underwater sound are reviewed for tactical consequences a pattern of sonobuoys is dropped with favourable spacing and depth settings. It does not take long before a contact emerges on one of the outer buoys.*

***2000.** An expeditionary force of various surface ships is about to enter a coastal area. To predict the performance of various acoustic sensors, the water column is sampled with a bathythermograph. Details about the local bottom conditions are unknown and the sonar performance model is then run with global parameters from an environmental database. As a result a mine hunting operation takes twice the time that was actually needed to clear the mines because of non-optimal sonar settings. Meanwhile a bottomed submarine remains effectively hidden in the reverberation, waiting for the main force to close in.*

***2010.** An amphibious force is to land on a beach that has been selected from satellite imagery. A discrete campaign of rapid environmental assessment then reveals the presence of a muddy sediment layer. Mud is ideal sediment for self-burying mines and means that the beach is not accessible for heavy armoured vehicles. With the secretly gathered information a new area is selected and the amphibious operation unfolds itself as an unopposed landing.*

Introduction

When expeditionary forces enter shallow or confined waters, the environment has a great influence on the performance of platforms, sensors and weapon systems. For this reason, environmental knowledge is regarded as one of the key factors in making decisions on the course of action and asset allocation [1]. The examples above illustrate how the right level of battle space information enables effective operational planning and mission execution [2]. For naval oceanography the main objective is to provide forces with a competitive advantage over adversaries by exploiting the current and future state of the environment. The Royal Netherlands Navy (RNLN) possesses various sensor performance models and tactical decision aids for its combat systems. Many environmental input parameters can be provided in advance by the Netherlands Hydrographical Office (nautical charting) and the METOC office of CODAM (environmental briefing dockets and databases). Some parameters are measured or sampled at sea, such as weather conditions, water temperature and underwater ambient noise. For expeditionary operations it is likely that *a priori* knowledge about the environment is limited and outdated. Therefore there is a need for tools that enable hydrographers or naval oceanographers embedded with the forces to collect and validate environmental information at sea.

Environmental information for naval warfare

Each mission type has its own operational need for environmental information in terms of data accuracy and spatial and temporal resolution [2]. In Anti-Submarine Warfare (ASW) it is crucial to know how well sonar performs. Environmental information enables the prediction of acoustic detection ranges on submarines and surface ships. For the open oceans, the propagation of sound is determined by depth, temperature and salinity only. Shallow waters are often characterized as an unpredictable and complex environment. For sonar, the performance is determined by many factors, such as tides, currents, wind, rain and reflections from the sea surface and complex bottom structures. The essential data for propagation modelling is often incomplete, and therefore the daily predictions of sonar performance are seldom close to reality. In addition, water conditions and sound speed profiles change during the day due to temperature changes and weather conditions. Mine Counter Measures (MCM) also depend on various oceanographic factors [3]. The bathymetry (charted water depth) and the acoustic properties of the medium determine how well mine hunting sonar will perform. Acoustic detection of mines is limited by sea bottom reverberation. A rough estimate of the sediment type is sufficient to indicate the underwater visibility and the likelihood of mine burial, but coastal mechanisms of river outflows and sediment transport call for repeated observations. In amphibious operations the shallow water bathymetry determines how close to the coast support ships and landing craft can safely get. Important information about the beach, such as trafficability and the slope, can be found with an autonomous underwater vehicle during high tide. In general, the characterization of the sediment and bathymetry for amphibious purposes permit a rough level of detail.

It is easily overlooked that the shallow character of the littorals can also be exploited. A rough approximation of the underwater battle space is already valuable for a tactical exploitation of the environment (TEE). A submarine can tactically exploit the reverberant properties of the sea bottom or be positioned to benefit from the directionality of ambient noise. TEE concerns easy rules of thumb and can do with rough estimates about the environment, as in "active sonar performs better in down slope direction then up slope". Environmental knowledge with a high level of detail enables passive source localisation with techniques known as Matched Field Processing (MFP). The advantage of MFP over conventional Doppler arithmetic is that the latter requires movement of the target and information about the zero-frequency and MFP does not. On the other hand MFP depends on a propagation model that operates on accurate environmental data. The technical character of MFP further calls for a highly skilled and well-instructed operator.

Various levels of battle space information can be obtained with a campaign of rapid environmental assessment (REA). The aim is then to measure, analyse and evaluate relevant properties of the environment in order to establish a recognized environmental picture (REP). The intention is that forces have a shared awareness of the battle space and that they have it in time. Since 2004 the RNLN operates two hydrographic survey vessels HNLMS Snellius and HNLMS Luymes. These modern ships are fitted with an extensive sensor suite for digital charting and further tasks of military hydrography [4]. For covert REA the navy may call upon Special Forces and submarines of the Walrus class, as was demonstrated during the exercise Joint Caribbean Lion (2006). Like many other navies within NATO, the RNLN is still in transformation from a blue water force to an expeditionary brown water force. Currently not all important environmental data for shallow water operations can (rapidly) be gathered.

Acoustic sensing in shallow water

The environmental factors that impact acoustic sensing capabilities are manifold. Shallow bathymetry and underwater obstacles may hinder the use of long towed arrays. The presence of divers or marine wildlife may call a halt to mid or low frequency sonar transmissions. Coastal ambient noise includes an abundance of directional sound sources with man made or natural origins. The focus of this paper is on those parameters that influence sound propagation, or more specific: the transmission loss due to sea bottom interaction. The water column is usually characterized by measuring conductivity (to estimate salinity) and temperature as function of depth (CTD sampling). Some empirical formula, e.g. in [5], is then used to calculate the sound speed profile. In deep water the propagation of sound is determined by this profile only; in shallow water many more parameters are involved. Various definitions can be given for shallow water [6]. From an acoustical point of view shallow water is found "when each ray from the source, when continued long enough is reflected at the bottom" [7, p9]. Another definition is "a water depth in which sound is propagated to a distance by repeated reflections from both surface and source" [8, p172]. To be practical, shallow waters are often said to be on the continental shelf and bordered by the 200 m contour line. Unlike the water column, the sea bottom cannot rapidly be characterized by insertion of some sampling device. Nevertheless, sound waves easily propagate in and out of marine sediments. Received signals can then be analysed with geoacoustic inversion techniques to backtrace acoustic properties of the ocean bottom from the spatial and temporal structure of sound pressure fields. Experiments for seabed assessment utilize a sound source and a receiver array for a one-time observation at sea of bottom reflected sound. A geoacoustic inversion process is then initiated to find a parametric description of an environmental model in terms of sediment layering properties and geoacoustic parameters such as sound speed, density and attenuation.

REA as a research project

The Rapid Environmental Assessment (REA) project at the Netherlands Defence Academy aims to understand the nature and impact of environmental conditions on the propagation of sound in shallow waters and sedimentary bottom types [9]. As such, the project aims for the development and validation of acoustic remote sensing systems and inversion methods. The result is a reliable and rapid environmental assessment of shallow water areas in support of various mission types. The question for this article is: what acoustic information about the seabed can be obtained from bottom-reflected shipping noise? The feasibility of geoacoustic inversion with non-traditional sound sources will be studied with data from two sea trials. During Saba'06, a Caribbean survey of the NL Hydrographic Office (NLHO) in 2006, small-scale experiments in a remote and isolated area were conducted from hydrographic survey vessel HNLMS Snellius [10]. The trials demonstrated a rapid deployment of sensors and equipment and resulted in a well-documented acoustic dataset. A unique achievement is that geoacoustic inversion was performed while the team was on board and an environmental debrief was provided, all within 24 hours. The BP/MREA'07 sea trials of 2007 were a much bigger effort [11]. Together with the NLHO, the NATO Undersea Research Centre (NURC) and various other institutions a shallow water area in the Mediterranean Sea was surveyed with a multitude of sensors. The overall aim of the trials was to demonstrate the concept of naval battle space preparation by providing a recognized environmental picture (REP). The dynamic and coastal area includes deeper water (200 m), very shallow water (30 - 10 m), a harbour approach and the beach. The multi sensor approach makes it possible to validate results of geoacoustic inversion experiments with non traditional sound sources under various circumstances.

Covert REA

The preparation of some remote coastal area with an overt REA campaign is in obvious conflict with the concealed nature of submarine and amphibious operations. Therefore environmental assessment in support of military operations will often be a discrete endeavour. Covert assessment of the sea bottom calls for clandestine deployment of sound sources and receiving sensors. The REA project studies various ways in which signals with geoacoustic information can be received. Receiving sensors can be inserted in denied areas by acoustic-oceanographic buoys and drifters that exploit the local currents [12]. A drifting buoy field covers a large area and is not hindered by the presence of mines, yet radio transmissions can be intercepted. During the scientific experiments Saba'06 and MREA'07, data was also gathered with a sparse vertical array deployed from a rubber boat. The concept can easily be translated to an operational context when acoustic-oceanographic sensors are deployed and recovered by Special Forces. The feasibility of this concept has recently been demonstrated with covert hydrographic reconnaissance during exercise Joint Caribbean Lion. More information about oceanography and Naval Special Warfare can be found in [13]. Front-line units such as autonomous underwater vehicles (AUV's) and submarines are already fitted with sensors for intelligence, surveillance and reconnaissance (ISR). Typical but sensitive intelligence missions can easily be extended with an environmental component to make dual-use of ISR sensors [2]. The approach also provides a capability to make dual use of *past* intelligence missions. In this case archived sonar data from ill-documented areas can be analysed again, but now for environmental purposes.

Sound sources of opportunity

For a thorough assessment of bottom properties acoustic signals are required with low frequencies that penetrate deep into the bottom. Shipping sounds are also low, with frequencies from 50 Hz up to 2 kHz. One of the reasons to launch a REA campaign is to aid in the prediction of passive acoustic detection ranges of ships and submarines. The conventional method relies on active sonar transmissions. There are however some practical down sides to the active approach. The high power consumption of low frequency systems limits the endurance of remotely deployed systems such as drifters, buoys and autonomous underwater vehicles [2]. And assessment with loud transmissions and low frequency is also more of an overt approach. An alternative is to utilize sound sources of opportunity. A military motive to do so is that (counter) detection is avoided and environmental assessment can be done in a discrete manner. Another motivation is that the method inflicts a minimal impact on divers and marine wildlife [10].

Coastal waters allow for a high concentration of human activities and as a result shallow waters are a noisy environment. With the right sensors there are many ships that can act as a sound source of opportunity. At some distance from the coast there is merchant traffic in designated shipping lanes, augmented by fishing vessels and offshore suppliers. Closer to the coast there are the ferries and the recreational boats. In times of military conflict various types of naval vessels may patrol coastal waters.

Figure 1. Sound sources of opportunity used so far in the REA project: HNLMS Snellius, NRV Leonardo, REMUS AUV and recreational boats

The REA project has lead to geoacoustic inversion with cooperative surface ships, unmanned underwater vehicles and even uncooperative recreational boats; the platforms are pictured in Fig. 1. For the Saba bank, geoacoustic inversion with received shipping noise from HNLMS Snellius revealed a very thin layer (15 cm) of sandy sediment over a sub-bottom of calcareous rock [10]. The BP/MREA'07 sea trials featured experiments with various sound sources of opportunity. When opportunities occurred, these sources behaved as planned, as in the experiments with self noise from HMLMS Snellius and NRV Leonardo [11]. During a particular run that focussed on the self noise from the relative quiet REMUS AUV [14] there was much interference from the weekend traffic. But then these recreational boats turned out to be fantastic sources of opportunity [15] and demonstrated the strength of the inversion method in using non-cooperative sound sources for a rapid and reliable characterization of the local sediment. In the following case study an AUV is used to assess the environment. The resulting environmental model is then demonstrated to enhance acoustic sensing capabilities with matched field source localization for one of the recreational boats.

Case study: geoacoustic inversion with an AUV

In a particular experiment during MREA07 a REMUS autonomous underwater vehicle was programmed for a mission in shallow water. An area was selected near the local harbour of Castiglione della Pescaia, Italy with a locally nearly flat bottom and a water depth of 33 m. The self noise of the vehicles was received on a sparse vertical array and used to invert sea bottom properties [14]. The general geometry of the experiment is pictured in Fig. 2.

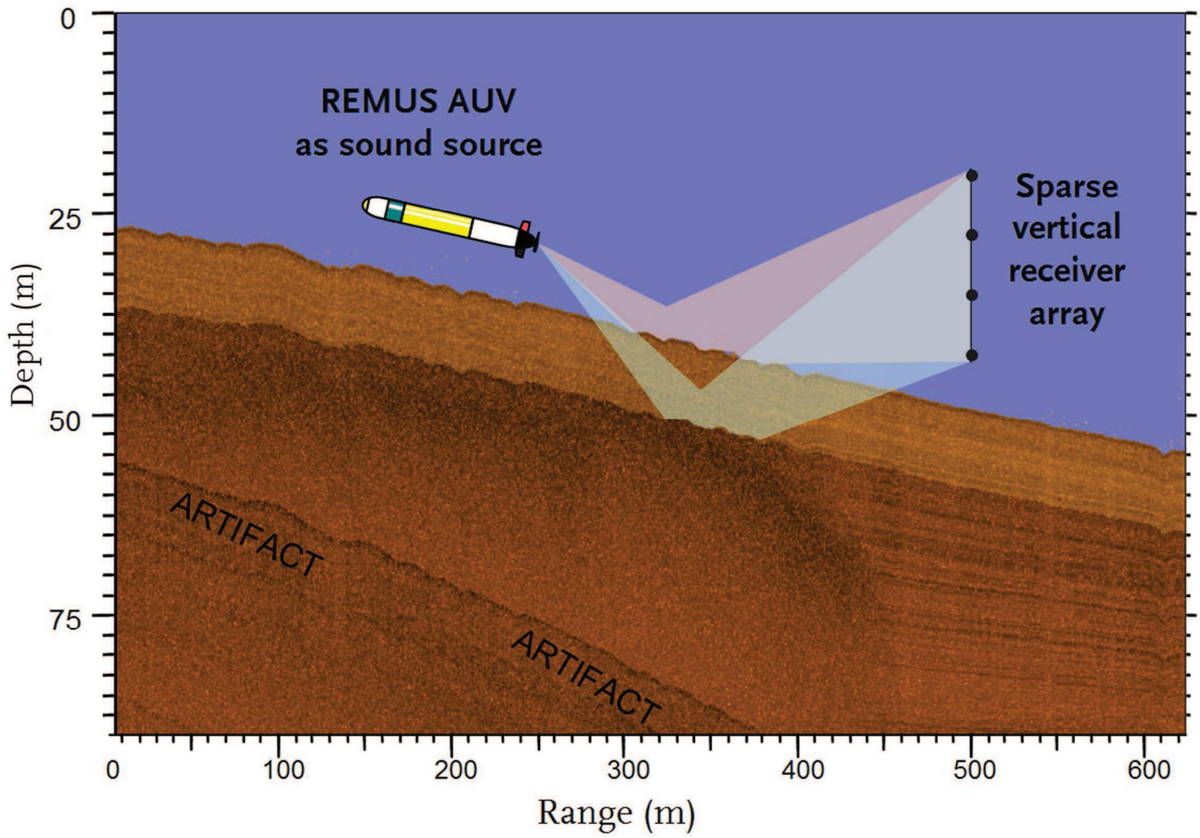

Figure 2. Concept of an inversion experiment where a REMUS AUV acts as a covert sound source of opportunity for geoacoustic assessment. Seismic profiling of the sea bottom by X-Star (TNO) on CD12500 line.

Methodology

Inversion is a search process for unknown acoustic parameters by comparison of observed underwater sound with replica data. A schematic overview of the process is given in Fig. 3.

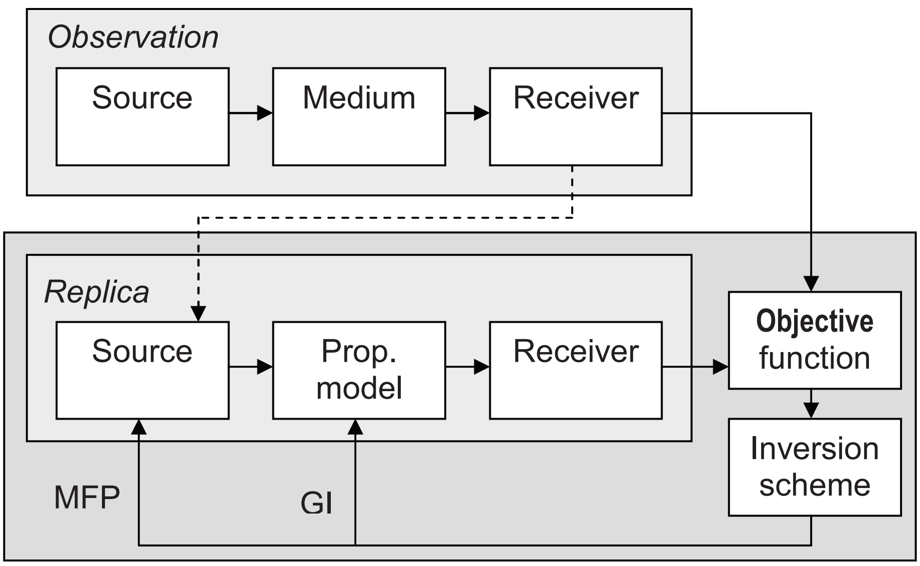

Figure 3. Diagram with the main components for Matched Field Processing (MFP) and Geoacoustic Inversion (GI)

Inversion begins with observations at sea when underwater sound is recorded. Further observations concern the experimental geometry and environmental data such as CTD samples to find the sound speed profile in the water column. Replica data with predicted propagation loss can be created with a propagation computer model. When a correct model of source position (MFP) and the underwater environment (GI) are input, the theory says that the output of the propagation model will be in perfect match with the observed data. Therefore in an iterative modelling process many input parameters are tested until a best fit is found. The mismatch is expressed by an objective function Φ. This is very often a Bartlett processor that cross-correlates data from a number of sensors [16]. The search strategy to minimize Φ in order to find an optimal solution is determined by an inversion scheme.

LOBSTER inversion toolbox

To carry out the inversion, a LOBSTER toolbox [17] has been developed at the NLDA (the Low-frequency Observation Based Sonar Toolbox for Environmental Reconstruction). This object-oriented Matlab code interfaces with variants of the KRAKEN [18] and MMPE [19] (third party) propagation models and offers a number of objective functions. The real innovation of the code is the support of inversion with acoustic particle velocity [20] and the number of included metaheuristic search strategies. Apart from conventional metaheuristics such as Simulated Annealing [16, 21] and the Genetic Algorithm [22], implementations of Differential Evolution [23] and Ant Colony Optimisation [24, 25] are included.

Geoacoustic inversion results

In the experiment, the AUV was programmed to run at its maximum speed and a ball bearing began to resonate. This proved to be highly beneficial as 8 stable tones were selected from a frequency range of 850 Hz to 1350 Hz. The phones in the sparse and vertical receiver array were at depths of 15, 20, 25 and 30 m. The applied inversion scheme was Differential Evolution with a population size of 50. The optimizer was configured to run for 40 iterations with a differential factor of 0.6, a crossover rate of 0.8 and a total of 1.6×10^4 calls to the KRAKEN propagation model. The used distances between source and receiver were less than 100 m [14].

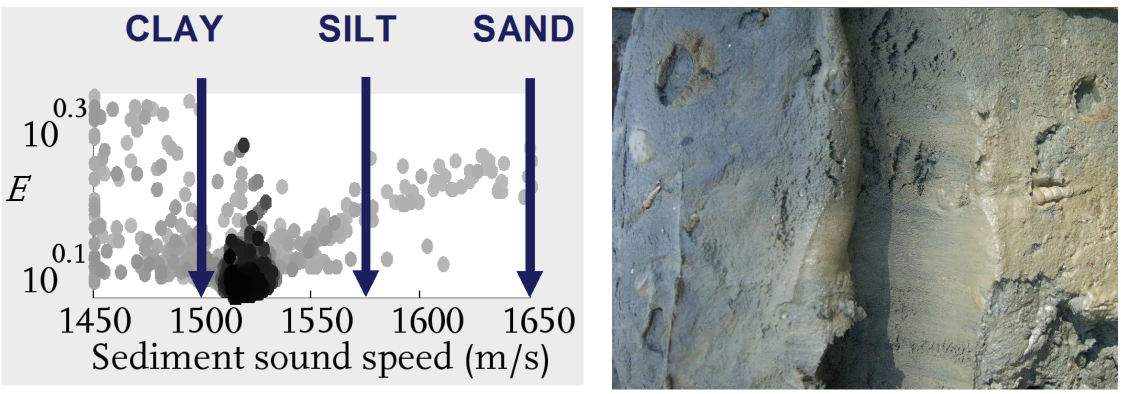

Figure 4. Convergence plot for the sediment sound speed. The markers for clay, silt and sand are from [26], the grab sample of silty clay was taken with a Van Veen grabber.

The dominant acoustic parameter turned out to be the sound speed of the sediment top layer. The 1520 m/s result obtained is characteristic for 'silty clay' [26] and corresponds with the grab sample from the sea bottom, both can be seen in Fig. 4. The seismic profile in Fig. 2 clearly shows the presence of a sub-bottom. In the inversion results however this sub-bottom was hardly perceived. A logical explanation, beside the low source level of AUV self noise, is that sound from the direct path and reflections from the sediment have considerable less propagation loss than the sub-bottom reflections. The direct path can be avoided by utilising downward reflection due to negative gradients in the sound speed profile. In this case geoacoustic inversion becomes more efficient with data from surface sources at greater distance from the receiver array.

Enhanced acoustic sensing

During the experiment there were many recreational boats that left Castiglione della Pescaia. Fig. 5 shows how one of these boats is localized with matched field processing for five tones from the inboard diesel engine, and given two different environmental models. When bottom properties from a military environmental database such as ASRAP are used, with a rough spatial resolution, the method fails to correctly identify the source position. MFP with the bottom model from the AUV inversion resulted in Fig. 5b with one clear spot at the surface and 920 m away from the receiver. This example clearly illustrates how proper environmental information enhances acoustic sensing capabilities.

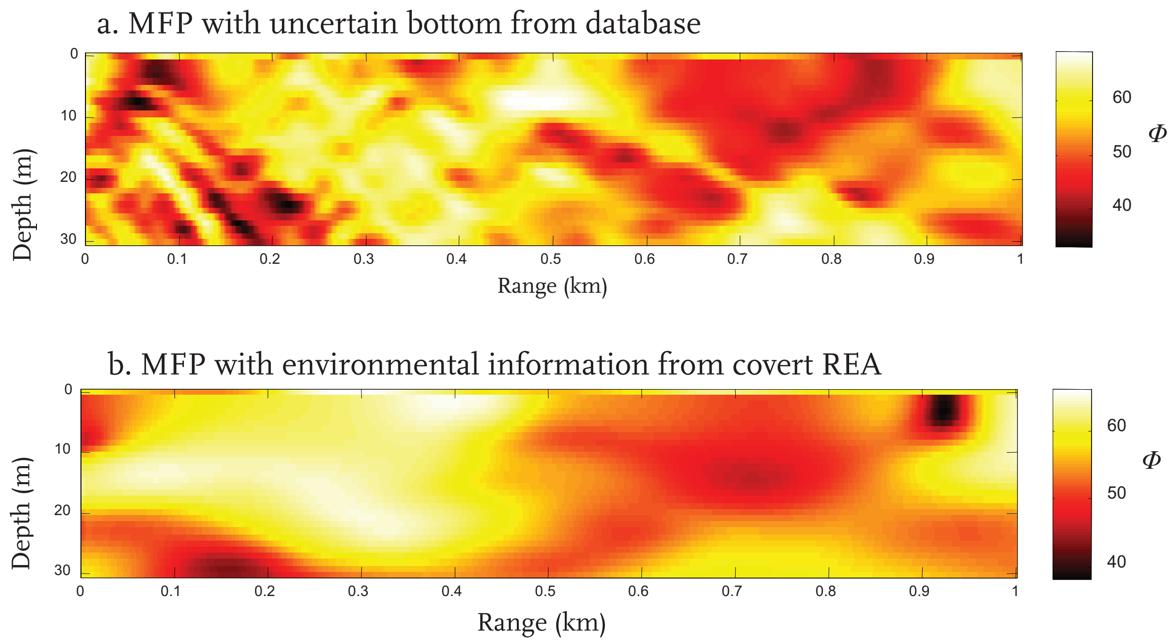

Figure 5. The benefit of environmental information for source localization with matched field processing. Pictured is the mismatch surface for depth and range. The engine noise of the recreational boat should be found just below surface, and is identified with minimal mismatch Φ, denoted with the colour black. The upper image (a) is based on uncertain bottom properties drawn from databases such as ASRAP and does not give a clear solution. The lower image (b) based on the covert REA mission with the AUV has one clear (black) detection of the recreational boat at the surface and 920 m from the receiver.

Discussion

Underwater acoustic sensing is a battle of the decibel. Quietening of submarines and increased ambient noise in coastal areas have resulted in a general decrease of acoustic detection ranges. For passive sonar in shallow water significant gains are possible when sensors with a vertical aperture are combined with modern signal processing techniques [27]. Real-time, environmentally adaptive algorithms may combine a track-before-detect approach with time-reversal algorithms in order to focus acoustic waves. Coastal ambient noise is highly directional in bearing and azimuth and this is where adaptive beamforming with arrays of directional vector sensors [28] can contribute even more. For passive sonar, environmental adaptive algorithms provide cleaner displays and easier track identification. The potential for active sonar is strong mitigation of reverberation. For expeditionary missions, relevant oceanographic data is often undersampled in space and time. Therefore, and to further adapt deep-water procedures for the littoral zone, the logical addition to XBT sampling of the water column is to assess seabottom properties with geoacoustic inversion techniques, as the U.S. Naval Oceanographic Office (NAVOCEANO) already practices [2]. The required resolution and acceptable level of environmental uncertainty depend on the range of mission types that naval forces fulfil. Significant advances in acoustic sensing are possible, yet they come with a price. Apart from the integration of dedicated shallow water sensors and environmentally adaptive processing, education and operational training remain a key factor. Acoustic sensing has never been easy, and a lack of education can easily degrade sensor performance. Then again, when the skilled hands of a 'techno sailor' are provided, major improvements in sonar performance are still possible.

Conclusions

The aim of this paper is to find out what acoustic information about the seabed can be obtained from bottom-reflected shipping noise. The feasibility of geoacoustic inversion with non-traditional sound sources has been studied with data from two sea trials. During the experiments on the Saba bank (2006) the concept was demonstrated with a short REA campaign in a remote and isolated area. With the MREA sea trials of 2007 in the Mediterranean Sea, the covert battle space preparation concept was further experimented with and complemented by a multi-sensor survey of various bottom types and water depths to further validate geoacoustic inversion methods. If there is one dominant parameter found that characterizes the sea bed in a shallow water area, it must be the sound speed in the upper sediment layer. With all inversions described here, the sediment sound speed was quickly found. This acoustic property prevails in its influence on the propagation of sound and it also identifies what material the seabed is composed of. Even a rough seabed characterization is highly beneficial for mine countermeasures as it suggests what mines can be deployed and indicates the possibilities and likeliness of mine burial. When visual and acoustic sensing capabilities are known it is possible to hunt mines in a time-efficient way. One step further is to use geoacoustic inversion to provide a full environmental model in support of antisubmarine warfare. When in situ data of high accuracy is input to a sonar performance model (such as Almost), instead of rough database estimates, the predicted detection ranges are guaranteed to be much closer to reality. It was further shown that geoacoustic inversion enables reliable remote sensing capabilities with matched field processing techniques. The proposed use of true sound sources of opportunity, such as ferries, recreational boating or military patrol boats, provides the navy with the capability of discrete rapid environmental assessment of remote and denied areas.

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