# On Line Measurement of High Frequency Ship Motions of Channel Bound Ships

*Dick Engelbracht & Wim van Buuren*

## Introduction

For the Dutch economy the ports of Amsterdam and Rotterdam are crucial. To continue the competitive positions of both ports versus foreign ports, the entrance channels to both ports have to be optimal. That said, it is however a problem. The largest ships calling at these ports have a very deep draught, so the ports are no longer accessible around the clock.

Still to receive such vessels, the competent authority worked with a so called tidal window. This means the ship steams at a certain time before the high tide (highest water level) and must have reached the port a certain elapsed time after the high tide. In other words there is an admission policy for the largest ships.

This has to do with the water depth of the fairway and the draught of the ship. This article discusses the draught of the ship. A ship sailing in waves has no constant draught but a varying draught due the ship motions, caused by the waves. These ship motions are: roll (rotation around the longitudinal axis), pitch (rotation around the lateral axis) and heave (translation in the direction of the vertical axis).

### Criticism of the current admission policy

Rijkswaterstaat calculates, based on a deterministic- (past) or probabilistic calculation method, the tidal window for a specific ship. This calculation, taking into account IMO regulations, included [1]: the predicted water level, the draught, the squat, a safety margin for the ship motions due to waves, the net under keel clearance, a margin for sedimentation between two soundings, sounding inaccuracies, extra dredged depth and the dredging tolerance. With these items you can calculate the net and gross keel clearance.

The master and to a certain extent the licensed maritime pilot, who advises the master about the navigation during the approach to the port area, will both be held responsible if the ship hits the bottom of the channel due the motions of the ship. Regarding the applied tidal window, they have to trust blindly on the calculations made by Rijkswaterstaat. That is, to say the least, not a desirable situation. The question arises: is it possible that the pilot during the transit across the channel can obtain an objective indication of the variation in the draught of the ship, including those resulting from the ship's motions by waves?

So this is a double edged sword. If it appears that the draught increase is larger than calculated, i.e. the tidal window is too wide, the passage should be aborted to avoid damage. On the other hand, if monitoring shows that the calculated tidal window is too short and that actually more time is remaining than calculated, this leads to a lengthening of the tidal window and hence an increase in the port accessibility.

## Measuring the ship motions

The answer to the above question can to some extent be positive. Give the pilot a set of sensors, especially accelerometers, place these sensors on board and determine the draught changes by monitoring these motions. If this is practical, still there are some disadvantages. The pilot comes on board by helicopter or by the pilot boat, often after the necessary obstacles meaning that he or she must have hands free and all luggage should be light, small and well manageable. The priority is the navigation of the ship, so equipment is installed and has to be up and running as quick as possible. With the introduction in 2005 of the Portable Pilot Unit (PPU), some things have been speeded up.

The PPU is an electronic sea chart in the form of a laptop connected to a GPS/GLONASS receiver. It is a very technologically advanced navigation tool, entirely independent of the navigation equipment on board of the ship. The PPU presents the pilot all the navigation information, clearly and with high accuracy, thus supporting him, along with his own observations, in his decision making process. Because use is made of a GPS/GLONASS receiver in a master-slave configuration, combined with real time position correction and a separate azimuth sensor, it is possible to calculate very accurately the position, the heading and one rotation angle of the ship. So this PPU has, at least partly, a sensor to determine the changes of the draught due to motions by waves.

Incidentally the Dutch Pilots in cooperation with the Netherlands Defence Academy has in the recent years developed a number of stand-alone programs which are installed on the laptop of the PPU, these are programs to calculate:

- Current forces on ships in confined channels and open sea [2];
- Anchor manoeuvring of vessels with very little under keel clearance;
- Forces and moments on the ship by wind;
- Squat [3];
- The anchor chain length, taking into account wind and current [4];
- Forces on mooring buoys;
- Wind forces on standard ship types [5].

### Conducted research

To investigate whether the PPU actually can contribute to an objective determination of the draught changes, a number of studies are being conducted, two of them by the Netherlands Defence Academy KIM. These studies are:

- To investigate the accuracy of angular measurement of the PPU, the angular measurement is compared with the angle measurement of two different inertial navigation systems. This study was conducted on a frigate of the Royal Netherlands Navy in the IJ-channel (IJmuiden) [6].

- In the Euro-Maas-channel (Rotterdam) a bulk carrier was equipped with four PPU distributed over the ship, so the three unknowns mentioned above, heave, roll and pitch, can be solved mathematically [7] and [8].

The purpose of both studies is twofold, first of all to investigate how the measurements of the PPU are influenced by the location of the equipment on board and secondly, whether it is possible with a PPU to measure the ship's motions.

### Outline of this article

This article does not explain the precise operation of the PPU. This is found in literature; see [6] and [9]. In section two, the underlying theory of motions of ship by waves is treated. Section three deals with the study of the bulk carrier. The article closes with section four, conclusions and recommendations.

## Mathematical and physical explanation about ship motions by waves

Ship motions can be considered as output of a system [10-12]. The input is the wave field, the output, as stated, the actual motions of the ship and the system, the ship itself. The output depends amongst other things on the ship's natural frequencies and the natural frequencies in turn depend on the dimensions and mass distribution of the ship.

### Description of the sea

Ship motions are a response to the resulting forces and moments which are exerted on the vessel. The resulting forces and moments in this case are caused by the sea. Sea can be seen as a superposition of many simple regular harmonic wave components each with its own period, phase angle, amplitude, length and direction of propagation, see Figure 1.

The properties of the single wave components should be known for more understanding of the wave field in which the ship is sailing. These properties, for instance the wave speed, include depending on the depth of the water [13]. Because we are dealing with harmonic signals, some advantage

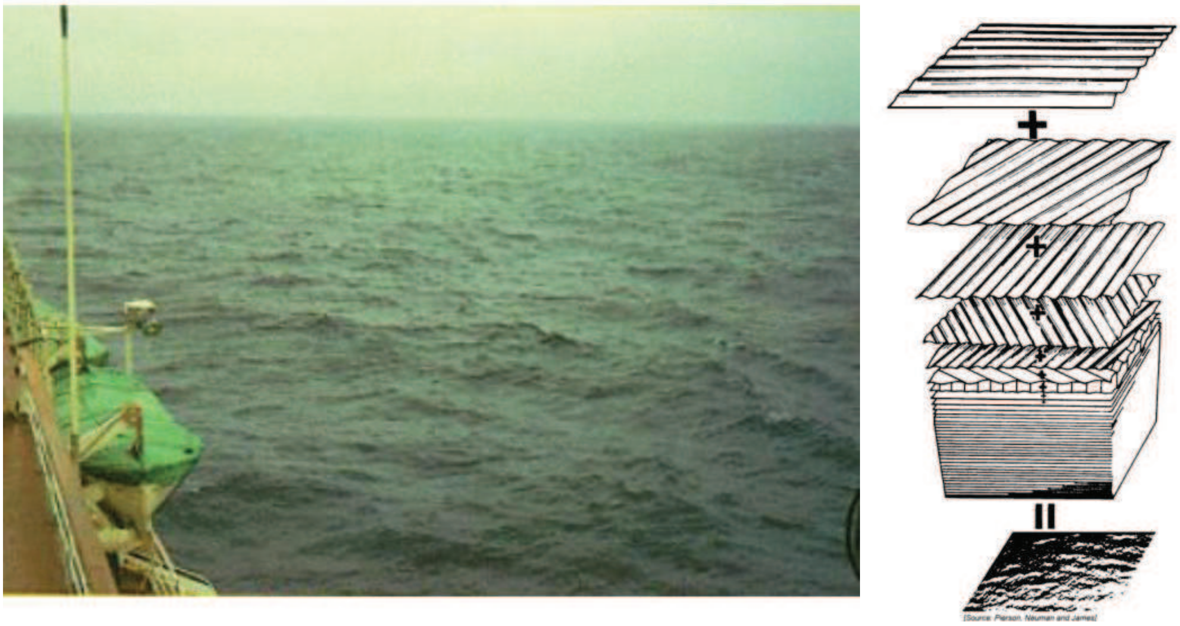


Figure 1: The surface of the sea left may be described by a sum of an infinit number of waves with different amplitudes, frequencies en directions (right). Both figures: source Delft University of Technology.

can be obtained from performing the calculations in the frequency domain, instead of in the time domain. With a Fourier analysis the time signal is converted to the frequency domain, and represented as a so-called wave-spectrum, see Figure 2. The wave spectrum is to be transformed into a so-called encounter-spectrum, because the ship is sailing with a particular course and speed compared to the speed and direction of the waves.

A wave spectrum can be regarded as a distribution of the square of the wave heights in the seas over the frequencies. The square of the wave heights is a measure of the energy in the waves.

It is obvious that, if the input is described in form of a wave-spectrum, the output should be described, in the same form as a motion-spectrum.

### Description of the ship motions

Three right-handed orthogonal coordinate systems are used to define the ship's motions:

- An earth-bound coordinate system $S(x_0,y_0,z_0)$. The $(x_0,y_0)$-plane lies in the still water surface, the positive $x_0$-axis is in the direction of the

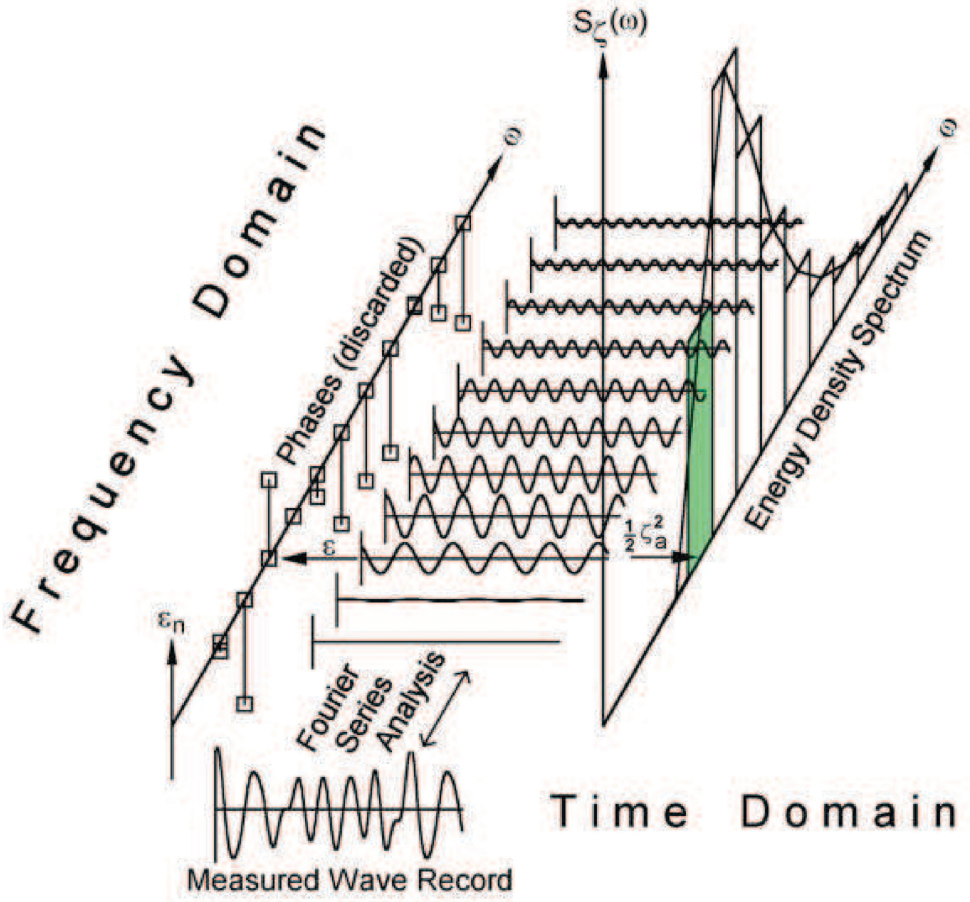


Figure 2: Conversion of a signal in the time domain to the frequency domain. Source: Delft University of Technology.

wave propagation. It can be rotated at a horizontal angle relative to the translating axis system O(x,y,z). The positive $z_0$-axis is directed upwards.

- A body-bound coordinate system $G(x_b,y_b,z_b)$. This system is connected to the ship with its origin at the ship's centre of gravity, G. The directions of the positive axes are: $x_b$ in the longitudinal forward direction, $y_b$ in the lateral starboard side direction and $z_b$ upwards. If the ship is floating upright in still water, the $(x_b,y_b)$ plane is parallel to the still water surface.

- A steadily translating coordinate system O(x,y,z). This system is moving forward with a constant ship speed V. If the ship is stationary, the directions of the O(x,y,z) axes are the same as those of the $G(x_b,y_b,z_b)$ axes. The (x,y)-plane lies in the still water surface with the origin O at, above or under the time-averaged position of the centre of gravity G. The ship is supposed to carryout oscillations around this steadily translating O(x,y,z) coordinate system.

Ship motions are defined as the motions of the ($x_b$,$y_b$,$z_b$) axes relative to the (x,y,z) axes. However, the PPU calculate the position in the ($x_0$,$y_0$,$z_0$) system.

A free moving mass has six degrees of freedom, three rotations: roll $\phi$, pitch $\theta$ and yaw $\psi$, and three translations: surge x, sway y and heave z. The height of any point n, ($x_b$(n),$y_b$(n),$z_b$(n)) on board, in the earth-bound coordinate system, mainly is influenced by the heaving, pitching and rolling motion of the ship. Ships are not totally rigid, so a seventh variable can be added, the vertical bending $w_z$ of the ship. The motions, x, y, z, $\phi$, $\theta$ and $\psi$ are defined as the motions of the mass centre of gravity G. Regardless of the transformations of the body-bound coordinate system to the earth-bound coordinate system, and what direction is positive or negative, the height of a point for instance the position of the antenna of a GPS/GLONASS receiver is equal to:

$$z_0(x_b(n), y_b(n), z_b(n)) = z + z_b(n) + x_b(n) \cdot \theta + y_b(n) \cdot \phi + w_z(x_b(n)). \quad (1)$$

This equation is linearized and only valid for small rotation angles. In the equation above, $z_0$($x_b$(n),$y_b$(n),$z_b$(n)) is measured by the master antenna of a GPS/GLONASS receiver at point n in the earth-bound coordinate system; $x_b$(n), $y_b$(n) and $z_b$(n) are the coordinates of a receiver in the body-bound coordinate system. In the equation there are basically four unknown variables: z, $\phi$, $\theta$ and the vertical bending $w_z$ at the point n. By using four receivers, the four unknown variables can be solved. By placing the master and slave antennas, in transverse or longitudinal direction, $\phi$ or $\theta$ can also be determined so the number of unknown variables became less.

## Research on board the bulk carrier Ferro Goa

During the run to the port of Rotterdam on board of MS Ferro Goa the next data of four PPUs were logged and written to a computer: time, position, angles etc. The passage to Rotterdam consists of two tracks, the ship starts on a track of 082.5° with a heading of 080° to compensate for current and wind. The second track of 112° is to be steered with a heading of 105°, halfway the passage. During the total passage time the sea state has remained roughly constant, this means that the ship has met waves from two different directions relative to the ship. In other words, two different encounter-spectra can be made.

## Details of the ship

In Table 1 one can see the loading condition of the bulk carrier Ferro Goa according to the onboard stability program.

Table 1: Loading condition of MS Ferro Goa.

| | | | | | | |
|---|---|---|---|---|---|---|
| $L_{ll}$ | 280.0 | [m] | GG' | 0.55 | [m] | |
| $L_{wl}$ | 286.0 | [m] | MG' | 5.27 | [m] | |
| B | 45.0 | [m] | $Y_G$ | 0.03 | [m] | starboard |
| $\Delta$ | 193938 | [ton] | $X_G$ | -7.43 | [m] | for the half length |
| $T_f$ | 17.42 | [m] | $T_0$ | 12.1 | [sec] | |
| $T_a$ | 17.58 | [m] | TPC | 121.26 | [ton/cm] | |
| $T_m$ | 17.50 | [m] | ETM | 2537.33 | [tonm/cm] | |
| KM | 18.73 | [m] | Ben.mom.(sag.) | -1645770 | [kNm] | 42% of the max. |
| KG | 12.91 | [m] | Shear force | -37897 | [kN] | 44.3% of the max |

## The locations of the PPU

Four PPU are placed on board. In Table 2 the coordinates of the PPU positions in [m] relative to G are in the body-bound coordinate system.

Table 2: Coordinates of PPU positions in the body system.

| | station | $x_b$ | $y_b$ | $z_b$ |
|---|---|---|---|---|
| PPU1 | one | 114.37 | -0.03 | 14.84 |
| PPU2 | two | -2.53 | -0.03 | 14.84 |
| PPU3 | three | -115.33 | 22.47 | 27.44 |
| PPU4 | four | -115.33 | -22.53 | 27.44 |

The PPU measures the heights in the earth-bound coordinate system, in addition, at station one and station four the pitch is measured and at station two and station three the roll. These rotations are not calibrated, nor to the earth-bound coordinate system nor to the body-bound coordinate system. The sampling frequency is equal to 5 [Hz].

## The test conditions

The test conditions are summarized in Table 3.

## The results of the measurements

By each PPU the height is measured and also at station two and station three the roll and at station four and station one the pitch. Some results can be seen for run one in Figure 3 and for run two in Figure 4.

Table 3: Conditions during the experiments.

| Dataset | | Run 1 | Run 2 |
|---|---|---|---|
| Length of the run | [hh:mm:ss] | 05:46:35-05:52:34 | 06:00:59-06:10:59 |
| Speed | [m/s] | 5.00 | 4.58 |
| Heading | [°] | 080 | 105 |
| Swell angle relative to the ship | [°] | 055 | 030 |
| Channel water depth | [m] | 24 | 24 |
| Swell direction | [°] | 315 | 315 |
| Water depth outside the channel | [m] | 17 | 17 |

Taking a closer look at the measured signals we can see that the measured heights of all four stations and the measured roll signal of station two have a small bandwidth and that the signals are little affected by noise. The noise on the signal from the roll of station three and the signal of the pitch of station one is just a little larger. The noise on the signal of the pitch of station four is by far the largest.

## The solution of the system of equations

As with four receivers, n1 to n4, are measured at the same time at four different locations on board, the unknowns z, $\theta$, $\phi$ and the bending can be solved out of a system of four equations, in matrix form the system of equation reads:

$$\begin{pmatrix} z_0(x_b(1), y_b(1), z_b(1)) - z_b(1) \\ z_0(x_b(1), y_b(1), z_b(1)) - z_b(2) \\ z_0(x_b(1), y_b(1), z_b(1)) - z_b(3) \\ z_0(x_b(1), y_b(1), z_b(1)) - z_b(4) \end{pmatrix} = \begin{pmatrix} 1 & x_b(1) & y_b(1) & 1 \\ 1 & x_b(2) & y_b(2) & \left(\frac{x_b(2)}{x_b(1)}\right)^4 \\ 1 & x_b(3) & y_b(3) & \left(\frac{x_b(3)}{x_b(1)}\right)^4 \\ 1 & x_b(4) & y_b(4) & \left(\frac{x_b(4)}{x_b(1)}\right)^4 \end{pmatrix} \begin{pmatrix} z \\ \theta \\ \phi \\ w_z(x_b(1)) \end{pmatrix} \quad (2)$$

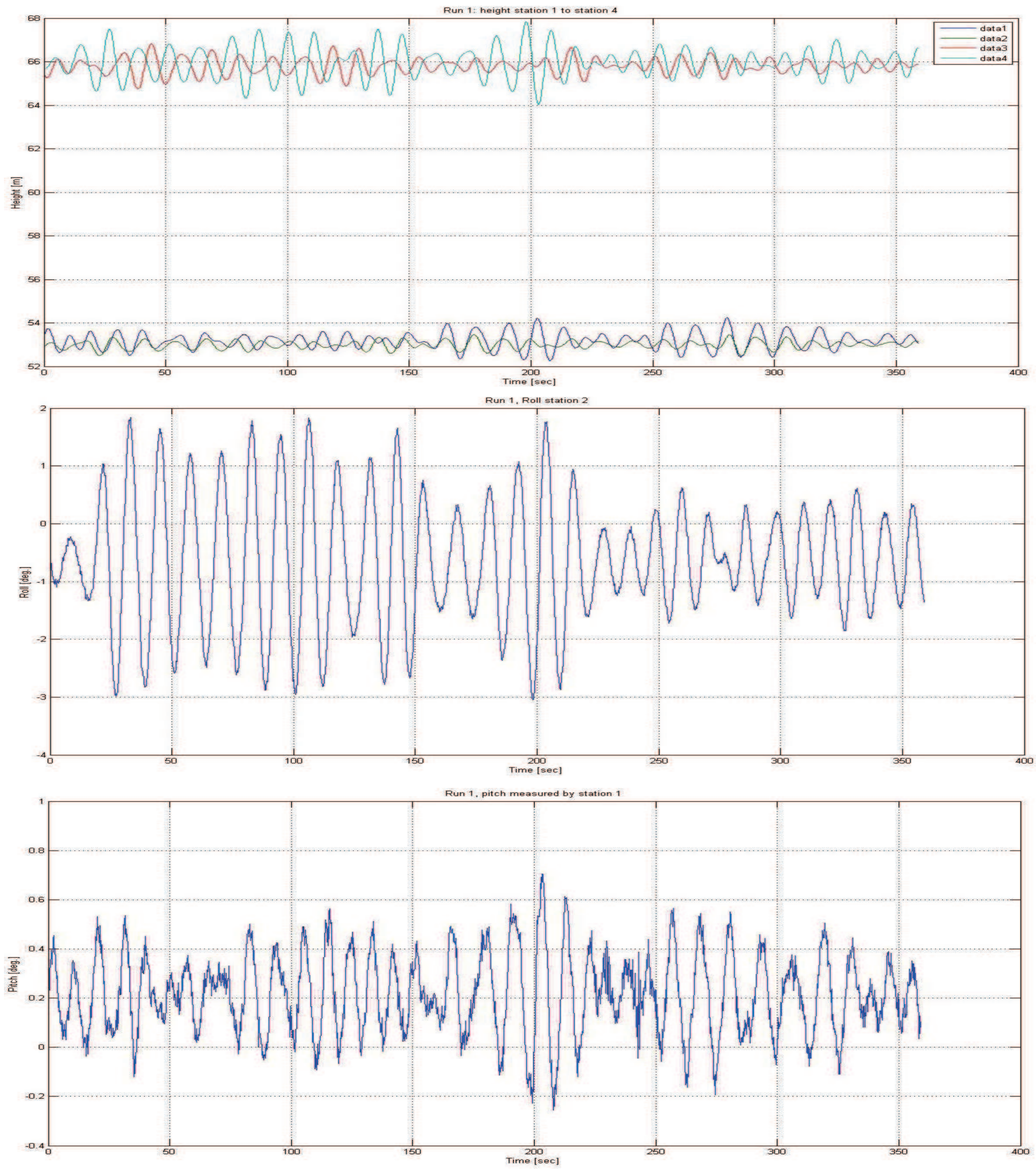


Figure 3: Run one, top: measured heights of the four stations; middle: measured roll of station two; bottom: measured pitch of station one.

The bending of the ship at various points can be determined by the bending at one point. This is based on common rules for the strength of materials:

$$w_x(x_b(n)) = \frac{x_b(n)}{x_b(1)}^4 \cdot w_x(x_b(1))$$

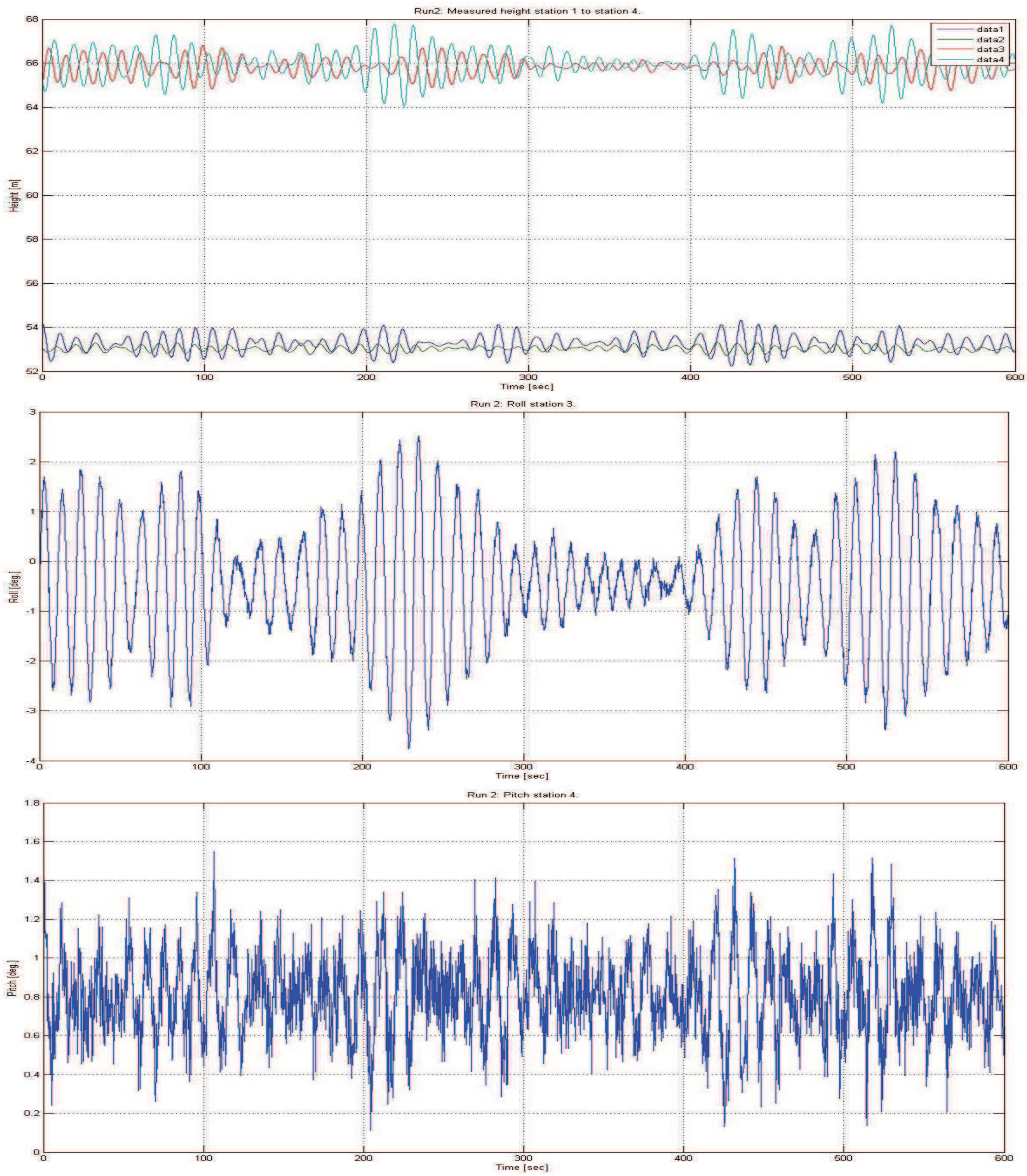


Figure 4: Run two, top: measured heights of the four stations; middle: measured roll of station three; bottom: measured pitch of station four.

On the left side of Formula 2 you can see the measured heights in the earth-bound coordinate system, corrected for the height of the antenna. On the right side of the system the first matrix includes the x and y coordinates of the receivers in the body-bound coordinate system and the expression for the bending of each location of the receivers. The second, most righthand matrix contained the unknowns to be solved.

To get a first impression about the accuracy of the solution compared to the measured roll and pitch of the various stations, a number of figures can be compared.

First, the solution of the heave motion z is easy to compare with the measured heights of station two, because the height measurement of station two is barely affected by the roll and the pitch of the ship. The reason for that lies in the fact that station two is located very close to the centre of gravity G, see Figure 5.

Secondly, the measured roll and pitch must correspond to the pitch ($\theta$) and roll ($\phi$) motion that follows from the solution of the system, see Figure 6.

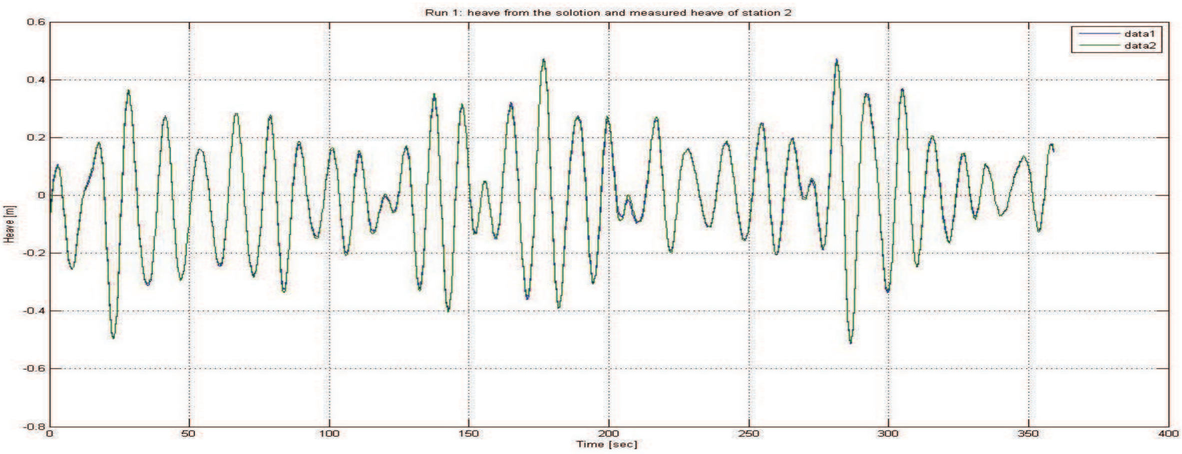


Figure 5: Run one, heave from the solution and the height measured by station two.

### Accuracy of the pitch and roll measurement

As said before, every station not only measured the height, but also an angle i.e. pitch or roll. At station one, and station four the pitch is measured and at station two and station three the roll. Unfortunately these sensors are not calibrated, neither in the earth-bound coordinate system nor in the body-bound coordinate system.

The mean values which can be calculated out of the measurements seem unrealistic. This is based on two considerations. First of all, after the ship is moored, the draught fore and aft are read and the values of the trim

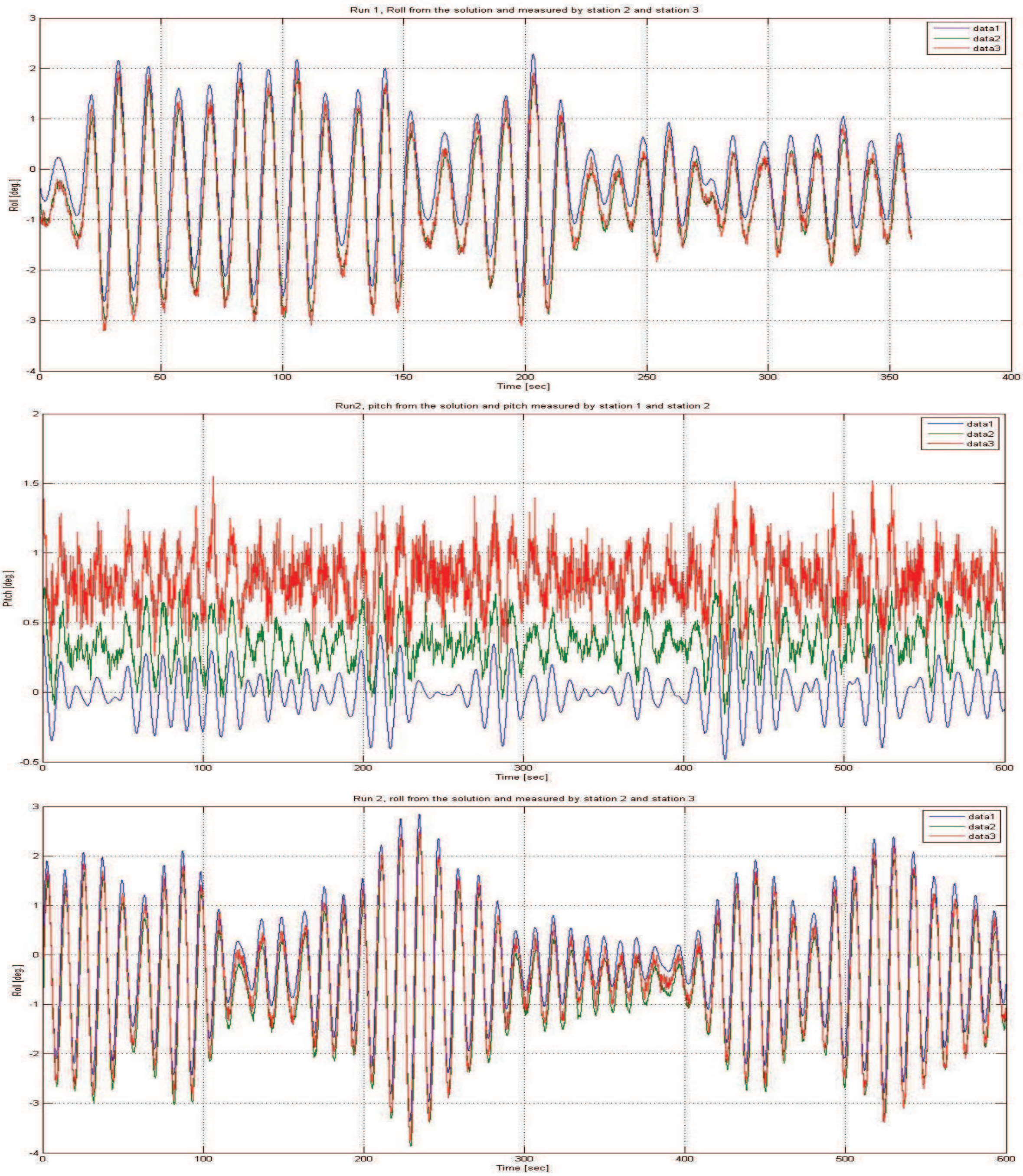


Figure 6: Top: Run one, roll from the solution and measured roll by station two and station three. Middle: Run two, pitch from the solution and the measured pitch by station one and station four. Bottom: Run two, roll from the solution and measured roll by station two and station three.

as a result of these draughts shows us a much lesser value than the mean pitch measured by station one and station four, secondly the leader of the experiments has a different feeling about the values, see Table 4.

Table 4: Mean values (in °) of rotations.

| run | Mean roll equations | Mean roll station 2 | Mean roll station 3 | Mean pitch from equations | Mean pitch station 1 | Mean pitch station 4 |
|---|---|---|---|---|---|---|
| 1 | -0.2 | -0.64 | -0.62 | -0.02 | 0.22 | 0.73 |
| 2 | -0.15 | -0.62 | -0.45 | -0.05 | 0.35 | 0.81 |

It is assumed that the list and trim angles that follow out of the solution of the system of equations (Formula 2) are more accurate than those which follow out of the measurements. The difference between the mean values is determined, a closer view is given to a possible phase shift, and last but not least, the ratio between the two amplitudes is determined.

To determine the accuracy of the angle measuring sensor, the time series of a motion, for instance the roll from station two, is given a little offset in time in relation to the reference signal as calculated by Formula 2. The mean for both signals is made zero, and the ratio $\alpha$ between the two signals is determined. An error is defined.

- $\zeta_{ref} = \alpha\zeta_{roll}$; the amplitude of the reference signal calculated with Formula 2 is a factor $\alpha$ multiplied by the amplitude of the measured signal.
- error $= \Sigma(\zeta_{ref} - \alpha\zeta_{roll})^2$; the total error equals the sum of the square of the values of the reference signal minus factor $\alpha$ multiplied by the value of the measured signal.

Table 5 gives the results for run 1.

Table 5: Comparison between the measured angles and the angles given by the system of equations, run 1.

| dt | 0.0 | -0.2 | -0.4 | -0.6 |
|---|---|---|---|---|
| Roll2, factor $\alpha$ | 0.9706 | 0.9868 | 0.9916 | 0.9849 |
| error | 0.0299 | 0.0094 | **0.0032** | 0.0114 |
| Roll3, factor $\alpha$ | 0.9447 | 0.9483 | 0.9408 | |
| error | 0.0141 | **0.0091** | 0.0188 | |
| Pitch1, factor $\alpha$ | 0.9103 | 0.9271 | 0.9303 | 0.9210 |
| error | 0.0018 | 0.0014 | **0.0013** | 0.0016 |
| Pitch4, factor $\alpha$ | 0.5393 | 0.5534 | 0.5586 | 0.5572 |
| error | 0.0062 | 0.0058 | **0.0056** | 0.0056 |

Given a closer look at these results we can conclude that the angle measuring sensors two tenths to four tenths of a second is slow compared to the reference signal. It also appears that the roll measured at station two, has a deviation to the reference signals of less than one percent and that the pitch measured at station four has the greatest deviation. The smallest value of the error is coupled to the maximum value of factor $\alpha$ (closest to one).

It is obvious that the overall error of the pitch is much smaller than that of the roll, this can be explained because the values of the pitch are about a factor five smaller than that of the values of the roll.

To visualize the accuracy of the angle measuring sensor both the signals are plotted. First the roll of the reference signal is plotted together with the measured signal of station two. The measured roll is corrected with the factor a, and the time shift. The same is done for the pitch of station four, both for run one, see Figure 7.

### Individual contribution to the draught change

Now we can consider changes of draught by the individual contributions of the ship motions. Starting point is that the ship is upright with a zero trim. Use is made of linearized equations without taking into account the bending of the ship. To look at the total motion of a certain point on the ship, a point on the ship is chosen (115, 22.5, -12.91), i.e. the point lies 115 [m] before the centre of gravity G, 22.5 [m] to starboard and 12.91 [m] beneath G:

$$z_0(115, 22.5, -12.91) = z + -12.91 + 115 \cdot \theta + 22.5 \cdot \phi$$

The average heave again is reduced to zero. Negative results indicate a sinkage so increasing draught. Positive outcomes indicate a rise or decreasing draught.

In Table 6 maxima and minima are shown by the individual motions and the total motion. In Figure 8 this is visualized.

Due to the fact that the phases and periods of the individual motions are not equal to each other, the maximum of the total is not equal to the sum of the maximums of the individual contributions, as is the minimum. You can see in Figure 8 that the contribution to the total motion by the pitch and the roll are of the same order and that the heave is about half of

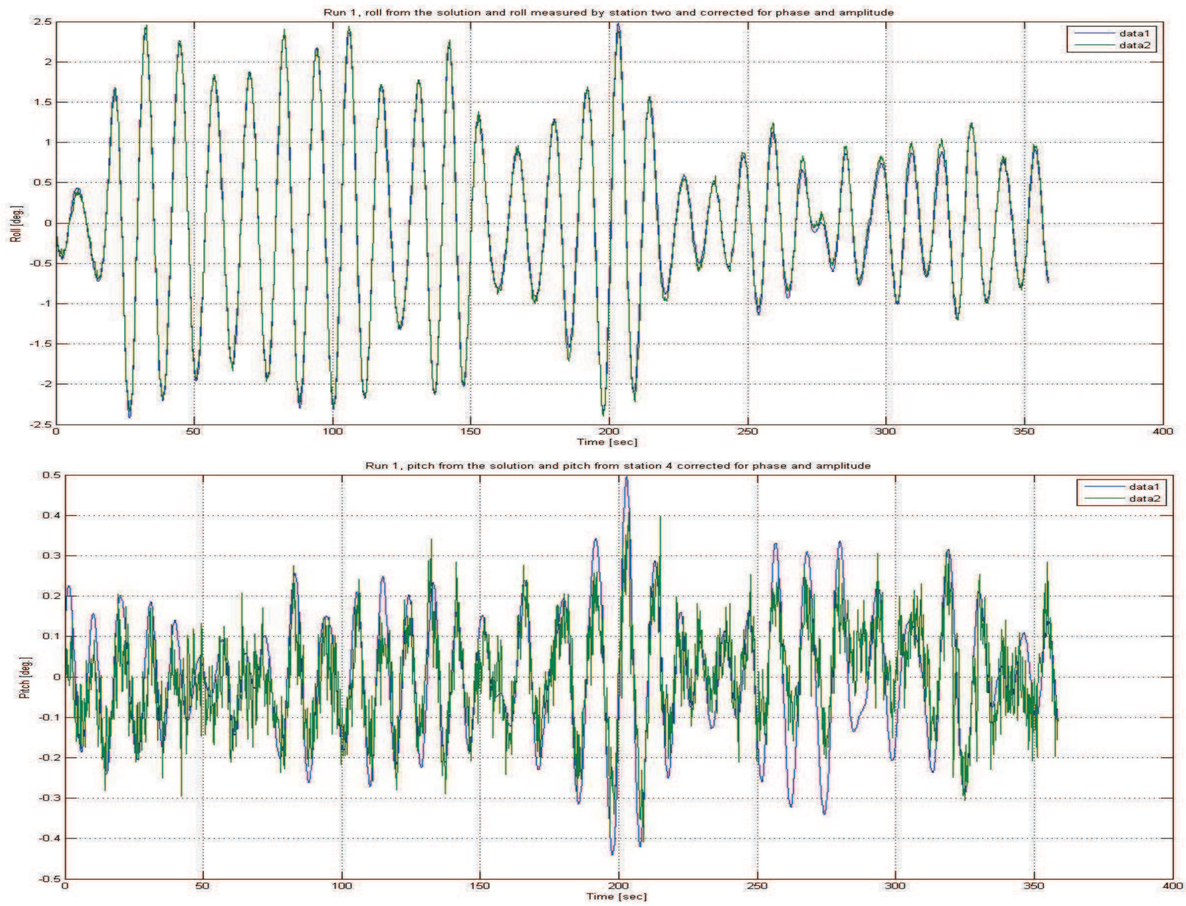


Figure 7: Top: roll from the system of equations compared to the roll measured by station two, the measured values are corrected for phase and amplitude (run one). Bottom: pitch from the system of equations compared to the pitch measured by station four, the measured values are corrected for phase and amplitude (run one).

Table 6: Individual contributions (in [m]) to the changes of draught by the motions of the ship.

| | | Heave | $115{\cdot}\theta$ | $22.5{\cdot}\phi$ | Total |
|---|---|---|---|---|---|
| Run 1 | max. rise | 0.47 | 1.00 | 0.97 | 1.93 |
| Run 1 | max. sinkage | -0.51 | -0.89 | -0.95 | -1.74 |
| Run 2 | max. rise | 0.29 | 0.94 | 1.18 | 1.78 |
| Run 2 | max. sinkage | -0.30 | -0.96 | -1.26 | -1.86 |

it. The light blue coloured line of data 4 represent the signal of the total vertical motion at point (115, 22.5, -12.91). By comparing run one with run two, it is seen that the heave motion decreases and the roll motion increases. The pitching remains more or less the same as the total motion.

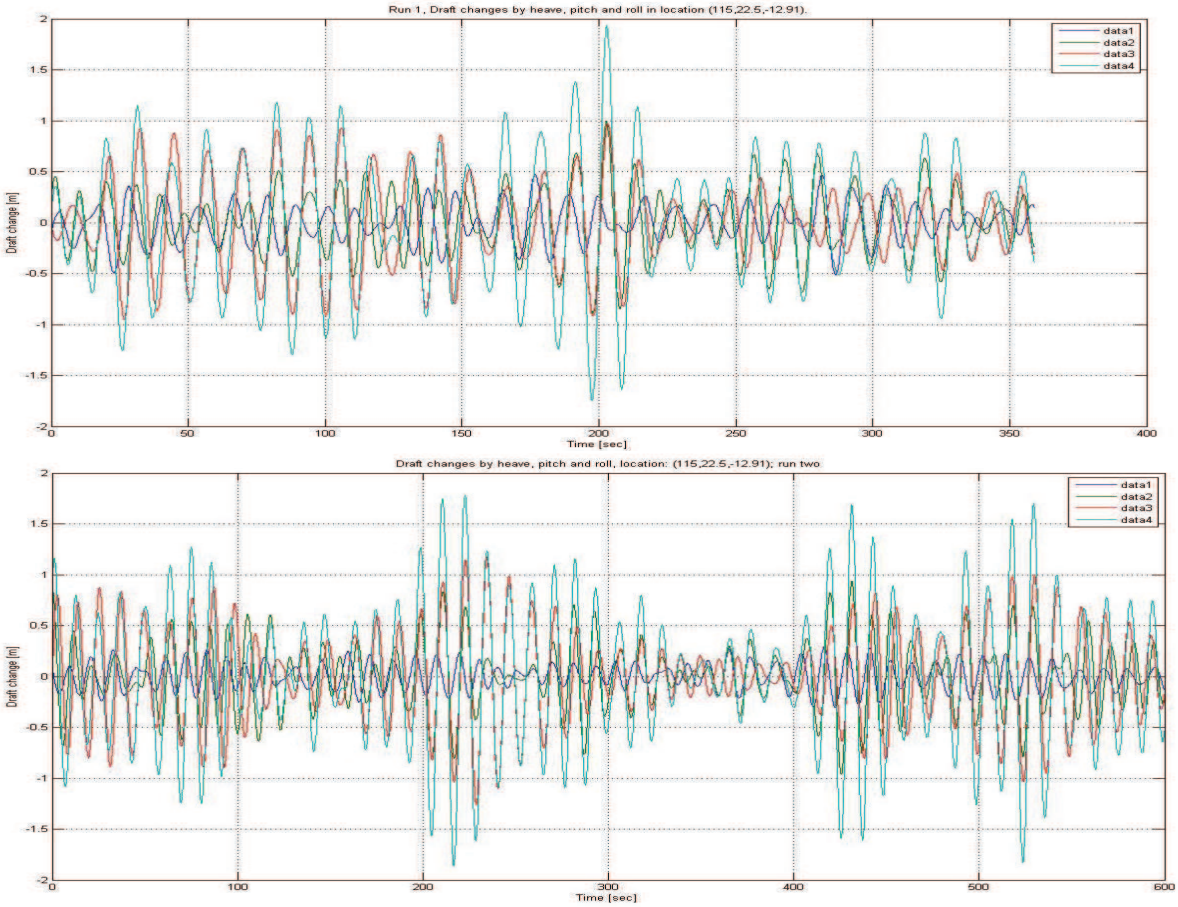


Figure 8: draught changes due ship motions during run one (top) and run two (bottom) respectively.

## An analysis in the frequency domain

The question remains whether the findings above can be substantiated based on theoretical considerations. For the roll motion the natural roll frequency of the ship should be considered. For the heave and pitch motions the natural frequency is less important, but what should be considered is the relationship between the apparent wavelength and the length between perpendiculars of the ship.

### Roll

If the encounter period between ship and sea is nearby the natural roll frequency of the ship 'resonance' can occur. In Figure 9 top, the wave-spectrum respectively the encounter-wave-spectrum of run one is shown. This encounter-spectrum is made based on the direction and speed of the vessel relative to the waves. A complicating factor is that the waterway depth is neither deep nor shallow. Figure 9 bottom shows the spectra of the roll motions from the solution of the system of equations, run two.

Table 7: The wave data from the original spectrum; H in [m], T in [s].

| $H_{significant}$ | $H_{mean}$ | $H_{maximum}$ | $T_2$ | $T_{visueel}$ | $T_{peak}$ | $H_{peak}$ |
|---|---|---|---|---|---|---|
| 2.05 | 1.28 | 2.61 | 5.3 | 5.8 | 9.1 | 0.52 |

Table 7 shows the statistical values of the original spectrum of the sea and the peak period. Table 8 shows the statistical values of the encounter spectra of run one and run two. Table 9 shows the statistical values of the roll spectra from the solution of the system of equations.

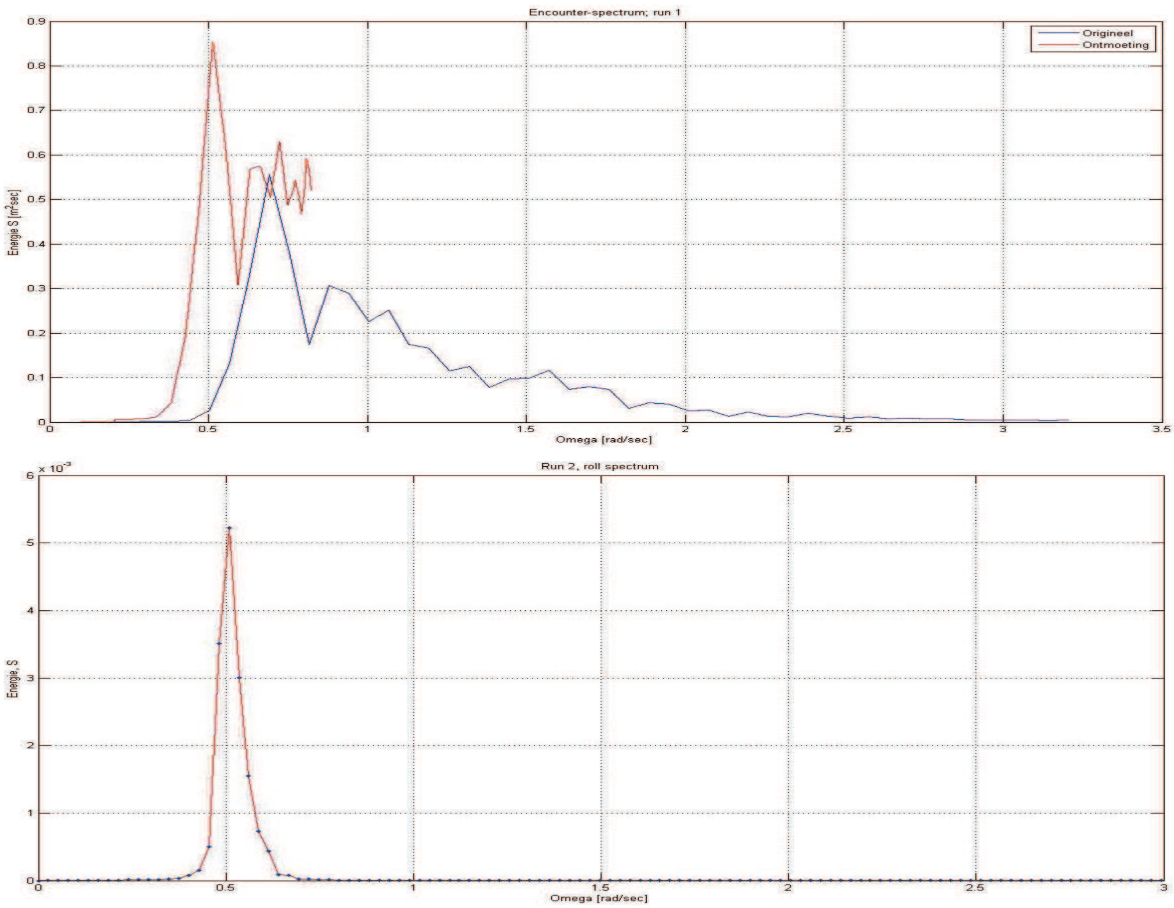


Figure 9: Top: encounter-spectrum, run one. Bottom: roll spectrum, run two.

Notably the spectrum of the roll is very narrow banded. A ship rolls more severely because the encounter frequencies of the waves excite the vessel close to the natural frequency of the roll motion. Here this is the case. The peak period of the encounter spectrum is equal to 12.2 [s] and

14.1 [s], while the peak periods of the motions are close by. Based on the figures the conclusion would be that during the second run the roll motion has to be slightly less intense, because the peak period of the encounter spectrum (14.1 [s]) is somewhat more on a distance relative of the natural period of the roll motion of 12.1 [s].

Table 8: Peak periods of the encounter spectra.

| Encounter spectra | $\omega_{peak}$ [rad/s] | $T_{peak}$ [s] |
|---|---|---|
| Run 1 | 0.513 | 12.2 |
| Run 2 | 0.4454 | 14.1 |

Table 9: Statistical values from the roll-spectra.

| Roll (solution) | $H_{mean}$ [°] | $H_{maximum}$ [°] | $T_2$ [s] | $T_{peak}$ [s] | $\zeta_{peak}$ [°] |
|---|---|---|---|---|---|
| Run 1 | 2.70 | 5.52 | 11.8 | 12.1 | 0.97 |
| Run 2 | 2.93 | 5.98 | 12.1 | 12.4 | 0.96 |

This is not the case, the motion is slightly increased, see Table 9. An explanation can be that the form of the stern is extremely flat shaped, the frames run relatively more horizontal. With oblique waves incoming to behind the ship, the ship is more asymmetrically lifted by the hydrostatical pressure and gets a more severe rolling motion.

From a physical and mathematical point of view the motion-spectrum is the result of the encounter-spectrum of the sea multiplied by the square of the transfer-function of the ship theoretically the transfer-function can be determined, but it is highly dependent on the hull form and the distribution of the masses. Software is required to get the transfer-function e.g. the program ‘Seaway’ in use by DUT.

**Pitch and Heave**

The determining factor for the strength of the pitch and heave motions is the ratio between the length of the ship and the apparent wave length. The ‘resonance’ peak lies near $(L_{ll}/\lambda)^{0.5}$ equal to one. In Figure 9 the original wave-spectrum and the wave-encounter-spectrum is shown. The left part of the spectrum until the first peak can be regarded as a representation of the swell in the wave field. This part is very narrow banded, typical fore swell. It can be deduced that the mean wavelength that belongs to this swell is equal to 100.8 [m]. The apparent wavelength is equal to 175.7 [m] for run one and 116.4 [m] for run two. The named quotient then is equal to 1.28 and 1.57 respectively, i.e. largely outside any resonance peak in the transfer-function. From Figure 8 you can see that the heave indeed is decreased, but the pitching remains more or less constant. Also this may be related to the shape of the ship.

For all three motions, roll, pitch and heave, from a physical and mathematical point of view, the strength of motions are equal to the square of the transfer-functions of the motions multiplied by the wave-encounter-spectrum. The transfer-function is highly dependent on the hull form and the mass distribution and can be determined on a theoretical basis (computer program).

## Determining the motion of the ship with one receiver

The main research question remains, is it possible to determine the motions of the ship, roll, pitch and heave, with one receiver, not taking into account the bending of the ship. Looking at Formula 1 three unknowns must be solved. If $\theta$ or $\phi$ is measured two unknowns are left, z and $\phi$ or z and $\theta$ and with one receiver it is impossible to solve these two unknowns. Maybe a different approach is possible by taking into account the equations for the horizontal motions of the ship. This gives two extra equations but three extra unknowns (x, y and $\psi$'). One of these unknowns is measured namely the course $\psi$', remains four unknowns ( x, y, z and $\theta$ or x, y, z and $\phi$). The following system of equations arises:

$$\begin{aligned}
x(x_b(n), y_b(n), z_b(n))' &= x - y_b(n)\psi' - z_b(n)\theta \\
y(x_b(n), y_b(n), z_b(n))' &= y + x_b(n)\psi' + z_b(n)\phi \\
z_0(x_b(n), y_b(n), z_b(n)) &= z + z_b(n) + x_b(n)\theta + y_b(n)\phi
\end{aligned}$$

Unfortunately these are three equations and four unknowns, so the system of equations is unsolvable. It should also take into account that some extra computation steps are required, i.e.: a correction for the current, a transformation of the earth-bound ($x_0$,$y_0$) coordinate system to the steadily translating coordinate system (x',y') and determination of the differences between the course and the mean course.

A next step could be that instead of one slave antenna, two slave antennas are used, one in the lateral direction of the ship to measure the roll and one in the longitudinal direction of the ship to measure the pitch. To determine the last unknown, the heave, principally we can use only the equation for the heights or the system of equations includes the equations for the horizontal motions of the ship. Whatever is preferable will be examined.

## Conclusions and recommendations

1. High-frequency ship motions are the motions of the mass centre of gravity G.

2. To determine the total sinkage of a point, the vertical displacement (heave), the roll and the pitch have to be determined.

3. The rotation sensors have to be calibrated relative to the body-bound coordinate system.

4. To estimate whether the motions of the vessel are reinforced or weakened, the wave-spectrum has to be transformed into an encounter-spectrum.

5. To determine whether the ship is responding, the quotient between the encounter period based on the dominant frequency of the waves and the natural period of roll has to be determined.

6. To determine whether the ship is responding, the quotient between the apparent wavelength of the dominant frequency of the waves and the length between perpendiculars of the ship has to be determined.

7. Most channel bound ships on their ways to the port of Rotterdam will find themselves in a sagging loading condition. This ensures that the draughts fore and aft will be less than in the midship. This is beneficial regarding the draught increased by motions, provided the starting point given the midship draught. Note that the draughts generated by stability programs usually do not take into account the bending of the ship.

8. With the current setup of the PPU, namely one receiver with one master and one slave antenna, it is not possible to determine all three components of the motions which affected the draught.

9. The position of the PPU on board seems to affect the angular measurement. Station two, near the mass centre of gravity, yielded the best results, station one and station four by contrast the worst.

10. Based on the results of the angle measuring determination of the roll seems to be more accurate than the pitch.

11. Examine the possibility to equip the PPU with two slave antennas, one in a lateral direction relative to the master and one in a longitudinal direction, making it possible to determine all three components of the motions.

12. Research which preferred approach to use: only the equation for the height, or a system of equations in which also the motions of the ship in the horizontal plane are taken into account.

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