MIMU PDR with bias estimation using an optimization-based approach

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***Abstract*—Traditional IMU based PDR systems suffer from rapidly growing drift effects due to the inherent bias of the inertial sensor. Many existing solutions to mitigate this problem use aiding sensors or information as heuristics or map data. We propose a new optimization framework to solve the PDR estimation problem where the sensors biases are explicitly included as state variables and therefore be used to correct for bias effects in the PDR. By using a smoothing approach and exploiting the rigid structure of a MIMU array one can solve for the slowly varying sensor biases. This paper presents the method and gives an exemplary result of a walking trial. Good agreements in the position and orientation with an optical reference system were found. Moreover, accelerometer and gyroscope biases could be estimated accordingly. Further research includes the performance of more experiments under various conditions such that a more quantitative evaluation can be obtained. In addition, an exploration of a (pseudo) realtime filter version would be valuable such that the system can be applied online.**

***Index Terms*—PDR, inertial sensor, IMU calibration, optimization, sensor bias, MIMU**

I. Introduction

The SaxShoe project aims to improve the safety of firefighters by tracking their positions and showing this information to the commander. To that goal, a small position tracking system is integrated in the boots of firefighters. To be able to track the position in all circumstances, e.g. indoor, Inertial Measuring Units (IMUs) can be used to estimate the position relative to the starting point without the need of GPS or any other physical infrastructure. MEMS based IMU Pedestrian Dead Reckoning (PDR) systems gained much attention due to the small form factor, low weight and relatively low cost.

The main problem of PDR is the accumulation of positioning errors due to erroneous sensor readings. Without any aiding information, position error quickly grow to several meters within a couple of seconds. Accordingly, reduction of errors is important for these PDR systems that has attracted a lot of research.

In this paper, we propose a solution to enhance the state and parameter estimation, based on IMU (accelerometer and gyroscope) data only. There are several techniques to address this problem. We will first give an overview of the existing techniques and subsequently present our proposed solution.

Rajagopal gave a sound description of PDR and proposed two solutions to reduce drift: Zero Velocity Update and Angular Rate Update [1]. Both methods rely on the assumption that the IMU is not moving during the stance phase of locomotion. At that moment, both the estimated translational and rotational velocities can be reset to zero. These techniques reduce the accumulation of errors significantly as the average time between two stance phases is often limited to less than a second.

During the stance phase one can also update the inclination of the foot by assuming that the accelerometer only measures the gravitational acceleration. However, the heading cannot be corrected by information from the IMU, hence those errors still accumulate due to a biased gyroscope sensor.

A possible solution to mitigate such errors is to introduce additional assumptions on the walking pattern. For example, the Heuristic Drift Reduction (HDR) method assumes that most hallways are straight and corners are 90 degrees [2].

Another approach is to include aiding sensors. MEMS IMUs are typically extended with a 3D magnetometer which can be used as a compass whenever the magnetic field is not disturbed. Outside one can apply GPS information to estimate the postion and velocity, whereas for indoor situations one can apply a map.

Most traditional approaches to PDR are based on Extended Kalman filters [3]. A more recent development is the use of optimization based approaches [4] which is more flexible as it can handle non-linearities and non-Gaussian noise sources.

The largest contribution to the position and orientation error of a PDR system is caused by, time varying, sensor biases. For example, gyro biases result in a drifting orientation estimate, whereas the accelerometer biases result in velocity and position drifting effects. Moreover, erroneous orientation estimate introduce an error the global inertial acceleration which is also referred as to gravity leakage.

In this paper we apply an optimization based approach to the PDR problem. The approach is demonstrated on a Multiple IMU (MIMU) platform which contains 4 identical IMU sensors that are rigidly mounted on a PCB. Such MIMU boards have several advantages. First, noise characteristics can be improved by averaging over the sensor readings. Second, by exploiting the relative poses of the IMUs additional information is obtained which assists in reducing dead reckoning errors.

The following sections present the proposed method. First the estimation problem is formulated as a cost function which includes the sensor models and the dynamics. Subsequently the experimental procedure is discussed and applied on a trial in which a rectangular trajectory has been travelled several times by a single subject. Finally, an exemplary result is presented to highlight the potential of our approach.

II. Method

The following set of variables (x) are included in the optimization framework from which eventually a point estimate will be calculated:

1) Gyroscope and Accelerometer biases:

$$\left\{b_{g,t}^{\mathrm{S}_i}, b_{a,t}^{\mathrm{S}_i}\right\} \qquad \forall \mathrm{S}_i \in \mathbb{S},\ \forall t \in \mathbb{T} \tag{1}$$

2) Body's kinematics:

$$\left\{p_{\mathrm{B},t}^{\mathrm{L}},\ v_{\mathrm{B},t}^{\mathrm{L}},\ a_{\mathrm{B},t}^{\mathrm{L}},\ q_t^{\mathrm{LB}},\ \omega_{\mathrm{LB},t}^{\mathrm{B}},\ \alpha_{\mathrm{LB},t}^{\mathrm{B}}\right\} \qquad \forall t \in \mathbb{T} \tag{2}$$

Where $b_{g,t}^{\mathrm{S}_i}$ and $b_{a,t}^{\mathrm{S}_i}$ are the sensor biases of the set ($\mathbb{S}$) gyroscopes and accelerometers respectively, at every moment in time ($\mathbb{T}$). The position $p_{\mathrm{B},t}^{\mathrm{L}}$, velocity $v_{\mathrm{B},t}^{\mathrm{L}}$ and acceleration $a_{\mathrm{B},t}^{\mathrm{L}}$ of the module (B) is expressed in a global static frame (L). Similarly, the orientation q_t^{LB}, angular velocity $\omega_{\mathrm{LB},t}^{\mathrm{B}}$ and angular acceleration $\alpha_{\mathrm{LB},t}^{\mathrm{B}}$ are defined with respect to the global static frame.

The following sections will explain the optimization framework and cost functions that are involved within the framework. This can be divided in state initializations, dynamic models and measurement models.

A. Optimization framework

The problem of estimating the the state vector $x_{t\in\mathbb{T}}$ can be formulated as Maximum A Posteriori (MAP) problem. We denote all the measurements, either real or virtual, by $y_{t\in\mathbb{T}}$. Now, the joint probability density function (PDF) is given by the product of the measurement's conditional density and the prior density. Considering the densities as being Normally distributed ($\mathcal{N}$) one can write:

$$p(y,x) = p(y|x)p(x) = \mathcal{N}_y(0,V)\mathcal{N}_x(\hat{x}^-, P^-)$$

The MAP results in the estimate of z that maximizes this function:

$$\begin{aligned}\hat{x} &= \arg\max_x \ p(y,x) \qquad (3)\\ &= \arg\min_x \ -\log p(y,x)\\ &= \arg\min_x \ -\sum_{t=1}^{N} \underbrace{\log p(y_t|x_t)}_{\text{measurement models}}\\ &\quad \underbrace{-\log p(x_1|y_1)}_{\text{dynamic initialization}} - \sum_{t=2}^{N} \underbrace{\log p(x_t|x_{t-1})}_{\text{dynamic model}}\end{aligned}$$

The solution of the MAP problem (3) coincides with the Weighted Least Squares (WLS) estimator if all probability density functions are modeled as Gaussians.

III. Sensor measurement models

A. Gyroscopes

The output of a gyroscope can be modeled as an angular velocity measured in sensor frame B with respect to an inertial frame L. We assume that the sensor S_i is rigidly connected to the underlying body, hence the relative angular velocity $\omega^{BS_i} = 0$. In addition we assume a bias $b_{g,t}^{\mathrm{S}_i}$ and Gaussian noise $e_{g,t}^{\mathrm{S}_i}$:

$$y_{g,t}^{\mathrm{S}_i} = R^{\mathrm{S}_i\mathrm{B}}\omega_{\mathrm{LB},t}^{\mathrm{B}} + b_{g,t}^{\mathrm{S}_i} + e_{g,t}^{\mathrm{S}_i}, \quad e_{g,t}^{\mathrm{S}_i} \sim \mathcal{N}(0,\Sigma^g) \tag{4}$$

where $R^{\mathrm{S}_i\mathrm{B}}$ is the relative orientation of the sensor (S_i) with respect to the underlying body (B) expressed as a rotation matrix. The bias is modeled as a first order Markov process :

$$b_{g,t+1}^{\mathrm{S}_i} = b_{g,t}^{\mathrm{S}_i} + w_{b_g,t}^{\mathrm{S}_i}, \quad w_{b_g,t}^{\mathrm{S}_i} \sim \mathcal{N}(0,\Sigma^{b_{g_i}}) \tag{5}$$

B. Accelerometers

The output of an accelerometer can be modeled as the sum of its experienced linear $a_{\mathrm{S}_i,t}^{\mathrm{L}}$ and gravitational acceleration g^{L} expressed in the sensor's coordinate frame S_i. In addition, a local bias $b_{a,t}^{\mathrm{S}_i}$ and Gaussian noise $e_{a,t}^{\mathrm{S}_i}$ term are included:

$$y_{\mathrm{a},t}^{\mathrm{S}_i} = a_t^{\mathrm{S}_i} + R^{\mathrm{S}_i\mathrm{B}}R_t^{\mathrm{BL}}g^{\mathrm{L}} + b_{a,t}^{\mathrm{S}_i} + e_{a,t}^{\mathrm{S}_i}, \quad e_{a,t}^{\mathrm{S}_i} \sim \mathcal{N}(0,\Sigma^a) \tag{6}$$

The linear acceleration can be expressed with respect to the bodies' acceleration assuming rigidity between sensor and body:

$$a_t^{\mathrm{S}_i} = R^{\mathrm{S}_i\mathrm{B}}\left(R_t^{\mathrm{BL}}a_{\mathrm{B},t}^{\mathrm{L}} + \alpha_{\mathrm{LB},t}^{\mathrm{B}} \times \mathrm{p}_{\mathrm{S}_i}^{\mathrm{B}} + \omega_{\mathrm{LB},t}^{\mathrm{B}} \times \left(\omega_{\mathrm{LB},t}^{\mathrm{B}} \times \mathrm{p}_{\mathrm{S}_i}^{\mathrm{B}}\right)\right) \tag{7}$$

where $\mathrm{p}_{\mathrm{S}_i}^{\mathrm{B}}$ and $R^{\mathrm{S}_i\mathrm{B}}$ are respectively the position and orientation of the accelerometer expressed in the body frame. Again, the bias is modeled as a first order Markov process :

$$b_{a,t+1}^{\mathrm{S}_i} = b_{a,t}^{\mathrm{S}_i} + w_{b_a,t}^{\mathrm{S}_i}, \quad w_{b_a,t}^{\mathrm{S}_i} \sim \mathcal{N}(0,\Sigma^{b_{a_i}}) \tag{8}$$

IV. Pseudo measurement models

A. Zero velocity

During stand-still we assume zero translational velocity:

$$0 = v_{\mathrm{B},t}^{\mathrm{L}} + e_{\mathrm{v}0,t}, \quad e_{\mathrm{v}0,t} \sim \mathcal{N}(0,\Sigma^{\mathrm{v}0}) \tag{9}$$

where $v_{\mathrm{B},t}^{\mathrm{L}}$ is the linear velocity and $e_{\mathrm{v}0,t}$ is a Gaussian error term.

B. Zero angular rate

Similarly, we assume a zero angular velocity during stand-still:

$$0 = \omega_{\mathrm{LB},t}^{\mathrm{B}} + e_{\omega0,t}, \quad e_{\omega0,t} \sim \mathcal{N}(0,\Sigma^{\omega0}) \tag{10}$$

where $\omega_{\mathrm{LB},t}^{\mathrm{B}}$ is the angular velocity and $e_{\omega0,t}$ is a Gaussian error term.

C. Zero height

We assume a flat walking surface and therefore that the vertical direction (z) during stand-still is zero:

$$0 = p_{\mathrm{B},z,t}^{\mathrm{L}} + e_{p_z0,t}, \quad e_{p_z0,t} \sim \mathcal{N}(0,\Sigma^{p_z0}) \tag{11}$$

where $a_{\mathrm{S}_i,t}^{\mathrm{L}}$ is the linear acceleration and $e_{p_z0,t}$ is a Gaussian error term.

D. Dynamic models:

The discrete kinematic equations predict the pose and velocity at $t+T$ given the current accelerations and sampling period T:

$$
\begin{aligned}
p^{\mathrm{L}}_{\mathrm{B},t+T} &= p^{\mathrm{L}}_{\mathrm{B},t} + T v^{\mathrm{L}}_{\mathrm{B},t} + \tfrac{T^2}{2}\left(a^{\mathrm{L}}_{\mathrm{B},t} + w_t^{p^{\mathrm{L}}_{\mathrm{B}}}\right)\\
v^{\mathrm{L}}_{\mathrm{B},t+T} &= v^{\mathrm{L}}_{\mathrm{B},t} + T\left(a^{\mathrm{L}}_{\mathrm{B},t} + w_t^{v^{\mathrm{L}}_{\mathrm{B}}}\right)\\
a^{\mathrm{L}}_{\mathrm{B},t+T} &= a^{\mathrm{L}}_{\mathrm{B},t} + w_t^{a^{\mathrm{L}}_{\mathrm{B}}}\\
q^{\mathrm{LB}}_{t+T} &= q^{\mathrm{LB}}_{t} \odot \exp\left(\tfrac{1}{2}\left(T\omega^{\mathrm{B}}_{\mathrm{LB},t} + \tfrac{T^2}{2}\left(\alpha^{\mathrm{B}}_{\mathrm{LB},t} + w_t^{q^{\mathrm{LB}}}\right)\right)\right)\\
\omega^{\mathrm{B}}_{\mathrm{LB},t+T} &= \omega^{\mathrm{B}}_{\mathrm{LB},t} + T\left(\alpha^{\mathrm{B}}_{\mathrm{LB},t} + w_t^{\omega^{\mathrm{B}}_{\mathrm{LB}}}\right)\\
\alpha^{\mathrm{B}}_{\mathrm{LB},t+T} &= \alpha^{\mathrm{B}}_{\mathrm{LB},t} + w_t^{\alpha^{\mathrm{B}}_{\mathrm{LB}}}
\end{aligned}
$$

where $\odot$ is the quaternion product operator and exp the quaternion exponential. The process noises are being described by:

$$w_t^{X^{\mathrm{L}}_{\mathrm{B}}} \sim \mathcal{N}(0,\Sigma^{\mathrm{X}}), \quad \mathrm{X} \in \{\mathrm{p},\mathrm{v},\mathrm{a},\mathrm{q},\omega,\alpha,\} \tag{12}$$

The dynamics of the sensor biases have been described in the sensor measurement model section.

V. Solving the total cost function

The total cost function is found by rewriting the individual stochastic models such that each is a function of the particular noise variable e or w. Subsequently, all functions are summed and weighted according its corresponding covariance.

This nonlinear least-squares (NLS) problem is solved using a Marquardt-Levenberg approach. To this end we used the Ceres library, which is a large-scale numerical optimization library targeting on solving bundle-adjustment problems [5] [6]. This library has several appealing properties such as handling large sparse matrices, using a proper quaternion parameterization and ability to perform automatic differentiation.

VI. Experimental method and results

The Inertial Elements MIMU22BT was used as a MIMU module, see Fig. 1. This module contains four Invensens 9205 inertial sensors and has the capability to read out the raw sensor data up to 500 Hz.

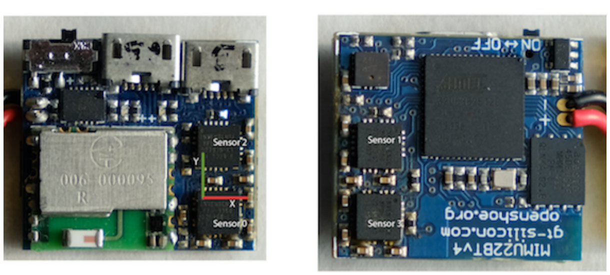

Fig. 1: Top and bottom view of the MIMU22BT. Visible are four Invensens 9205 modules. Sensor 0 is mirrored with sensor 3 and sensor 1 is mirrored with sensor 2. The right-handed coordinate frame of sensor 0 is indicated (x=red, y=green). The relative distance between sensor 0 and 2 is 6.3mm.

To demonstrate the feasibility, a subject was instructed to walk repeatedly along a rectangular path (3.5 x 1.5m) while wearing a firefighters' boot with the sensor module embodied in the heel, see Fig. 2. In addition, reflective markers were attached to boot which enables the reconstruction of the path using an optical system. The subject walked for approximately 2 minutes such that the rectangular path was traversed 10 times.

Fig. 2: Subject wearing the firefighter boots with embodied MIMU module. Data is transmitted to a laptop via USB. The trajectory is outlined with white adhesive tape on the floor. In addition, optical markers and cameras of the reference system are visible.

The inertial sensors were calibrated prior to the experiment using the method described by Tidaldi et.al. [7]. A time varying offset has been added to the sensor readings after the recording to explicitly demonstrate the ability of estimating inertial sensor biases. To demonstrate the robustness of our algorithms, artificial values are chosen to be large compared to common sensor biases and are slowly varying over time according a linear trend.

Preprocessing of the data included the extraction of static intervals using a Generalized Likelihood Ratio Test (GLRT) described by Skog et.al. [8]. After running the smoother, data of both measurement systems was aligned, both temporal as spatial, for comparison purposes.

A. Results

An example reconstruction of the path is depicted in figure 3. Indicated are the position estimates obtained from the optical (blue dots) and optical reference (grey). The zero (angular) velocity and zero height measurement updates are indicated with red dots.

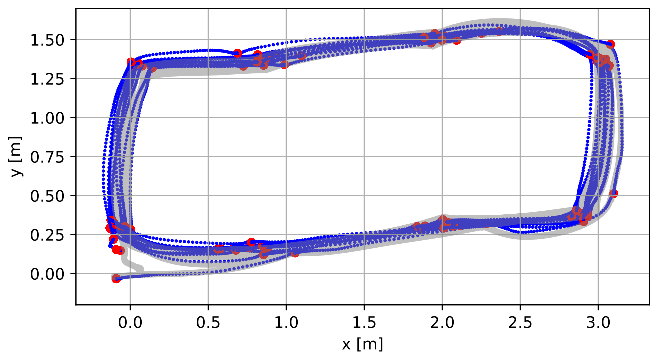

Fig. 3: Reconstruction of an example trajectory. Visible is the planar position estimated with the IMU system (blue dots) and measured with the optical reference system (grey). In addition, ZUPT periods are indicated with red dots.

Fig. 4 illustrates the differences in orientation and position between the inertial and optical system.

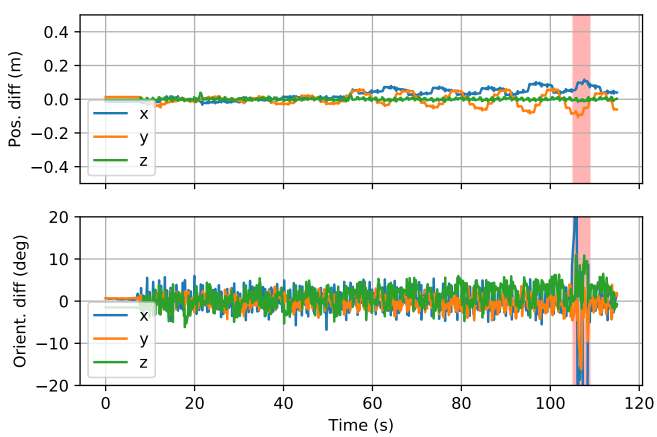

Fig. 4: Differences in the optical and inertial measurement systems. Visible are the position (upper) and orientation (lower) as function of time.

Finally, the ability of estimating sensor biases in depicted in Fig. 5. For each inertial sensor the (artificial) bias of the gyroscope and accelerometer is plotted.

VII. Discussion and Conclusion

Preliminary results demonstrate a method to solve for the bias problem in a PDR system. As a work in progress contribution this is a first attempt to show the potential of using an IMU array with an optimization approach. Latter allows for the incorporation of large amounts of data to ensure the observability of various states. However, the current method is implemented as an offline approach and therefore not suitable for online tracking of persons. Yet, adapting our implementation to an online method should feasible by applying a sliding window approach.

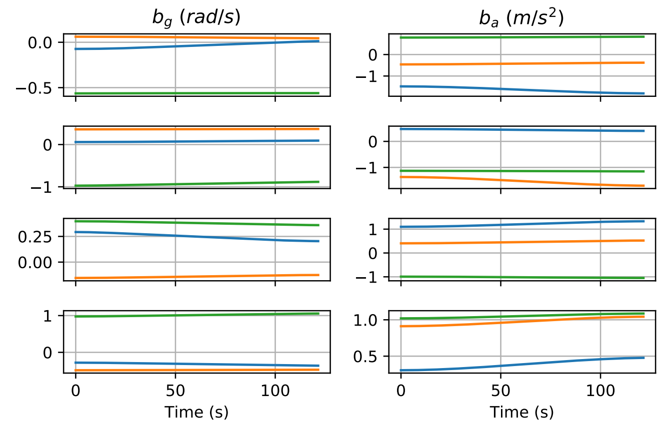

Fig. 5: Estimated sensor biases for each gyroscope (left) and accelerometer (right) pair. For each sensor the x (blue), y (orange) and z value (green) is indicated.

MIMU arrays are interesting as they can be used to improve the noise characteristics as well as providing new information due to the known structure of accelerometer and gyroscope on a rigid body. However, these advantages have only been investigated minimally so far. The conditions required for observability as well as sing such arrays to detect for sensor clippings requires additional research.

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