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Care on demand in nursing homes: a queueing theoretic approach

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Abstract Nursing homes face ever-tightening healthcare budgets and are searching for ways to increase the efficiency of their healthcare processes without losing sight of the needs of their residents. Optimizing the allocation of care workers plays a key role in this search as care workers are responsible for the daily care of the residents and account for a significant proportion of the total labor expenses. In practice, the lack of reliable data makes it difficult for nursing home managers to make informed staffing decisions. The focus of this study lies on the 'care on demand' process in a Belgian nursing home. Based on the analysis of real-life 'call button' data, a queueing model is presented which can be used by nursing home managers to determine the number of care workers required to meet a specific service level. Based on numerical experiments an 80/10 service level is proposed for this nursing home, meaning that at least 80 percent of the clients should receive care within 10 minutes after a call button request. To the best of our knowledge, this is the first attempt to develop a quantitative model for the 'care on demand' process in a nursing home.

Keywords Nursing homes · Random demand · Queueing theory · Service level (of response times) · Care on demand · Unscheduled care

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1 Introduction

The Western world is facing an aging [26] and, in many cases, a more dependent population [28]. Consequently, the demand for long-term care is increasing and expected to increase further over the next couple of decades [6, 12]. Moreover, the provision of long-term care is becoming more complex as the prevalence of multimorbidity increases with age [35]. Keeping pace with this rising, and increasingly complex, demand has become a central issue for policy-makers in both the U.S. and the countries of the European Union [6, 7, 11, 12]. Under pressure of these developments, long-term care facilities face the challenge of providing high-quality care whereas budgets do not increase at the same pace, or often even decrease.

Nursing homes are an important component of the long-term care system for elderly people with disabilities [31]. Most nursing homes are searching for ways to further streamline their (health)care and support activities[1] with the purpose of lowering costs while maintaining an appropriate quality level of care. From a client-centered perspective, in which the client's needs and wishes are the starting point for the delivery of care [2], 'quality of care' can be defined as the extent to which needs and preferences of the nursing home residents are being met [25]. According to Moeke et al. [24] an important aspect of client-centered care is that nursing home residents do not want to adjust their lives to the schedule of care workers, but want to have influence on the moment (day and time) at which care will be delivered. In practice, nursing homes face ever-tightening financial constraints. Hence, they have to balance the need for client-

[1] From now on we use the term 'healthcare activities' instead of '(health)care and support activities'.

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centered care with a more efficient use of resources. As care workers are the most prominent resource, staffing decisions play an important role in the search for more efficiency.

When it comes to healthcare delivery in a nursing home environment, a distinction should be made between two types of healthcare activities. For some of the care activities it is possible, based on the needs and preferences of the residents, to make a fairly detailed planning in advance. Examples of this type of activities are 'giving medicine' and 'help with getting out of bed in the morning'. These activities can be defined as 'care by appointment'. On the other hand, there are healthcare activities which are carried out in response to random, unexpected demand such as assistance with toileting. This type of activities can be defined as 'care on demand'. The focus of this study lies on 'care on demand' activities.

A considerable part of the 'care on demand' activities in nursing homes consists of responding to requests of nursing home residents made through the use of call buttons. Most nursing homes struggle with determining the appropriate number of care workers needed to respond to these call button requests. The main reason for this is that a well-founded quantitative approach is generally lacking. Staffing decisions concerning 'care on demand' are often made without a sound rational basis. In the ideal situation staffing decisions are based on a quantification of the needs and preferences of the nursing home residents, in other words the demand, and the duration of the healthcare tasks associated with these needs and preferences. Unfortunately, in most nursing homes this type of information is not available. Even basic staffing information as information about actual staffing hours is often of poor quality [18] or not available at all [17]. Our experience is that the lack of reliable data is a common problem in healthcare facilities. However, it is a more pronounced problem in nursing home facilities as they are often low-tech, paper based organizations.

In this paper a queueing model is developed, using data of a Belgian nursing home, to gain more insight in the 'care on demand' process and its performance. Thereby, this study provides better understanding of the number of resources (i.e. the number of care workers) required to sufficiently meet the needs of the nursing home residents regarding 'care on demand'. We analyze demand patterns over the course of a day, whereas our main focus is on the night shift. The reason for this is that the 'care by appointment' activities are scarce during the night, which minimizes the risk that the 'care on demand' data is compromised by 'care by appointment' tasks.

Clearly, this study addresses an issue of great societal relevance. More specifically, in order to increase the efficiency without loosing sight of the needs of residents, it should be possible for nursing home managers to analyse and monitor the performance of healthcare processes. From a scientific point of view there is hardly any insight in demand processes in nursing homes. To the best of our knowledge this is the first endeavor to study 'care on demand' activities in a nursing home setting using a queueing theoretic approach.

The paper is structured as follows. In the next section, we outline and justify the study by looking at related literature. In addition, we describe the nursing home context and its relation to queueing theory and propose a performance measure for 'care on demand' in nursing homes. In Section 3, we analyze the 'care on demand' data of a single Belgian nursing home. Based on this analysis we present a queueing model in Section 4. In Section 5 the constructed queueing model is used to analyze different scenarios. Finally, in Section 6, we present our conclusions and directions for future work.

2 Nursing home context

In this section the 'care on demand' process is described in more detail. First, we justify the use of a queueing theoretic approach. Next, we provide more insight into the empirical context regarding the 'care on demand' activities in a Belgian nursing home. In the final subsection we define a performance measure which can be used for assessing the 'care on demand' process.

2.1 A queueing theoretic approach

Waiting lines or queues occur whenever the demand for a service exceeds the system's capacity to provide that service. From a client perspective, long queues have a detrimental effect on the perceived quality of service. Unfortunately, congestion is commonplace in many areas of healthcare and has become an important issue in the provision of healthcare services. In addition to diminished patient satisfaction, waiting can have a serious impact on the well-being of patients or clients. In the case of nursing home residents, excessive waiting for care and/or support limits their freedom of living the life they prefer as they are often in need of ongoing assistance with activities in daily living.

Queueing theory is the mathematical study of waiting lines or queues and can be useful in describing and analyzing healthcare processes [16], where 'healthcare process' refers to a set of activities and/or procedures that a client takes part in to receive the necessary care. A growing number of studies shows that queueing models can be helpful in assessing the performance of healthcare processes in terms of waiting time and utilization of critical resources [10, 22]. The most common resources in a healthcare process are physicians, care workers, beds, and (specialized) equipment. When it comes to the delivery of healthcare in a nursing home setting, care workers can be regarded as the most important resource as they are responsible for the daily care and supervision of clients. Furthermore, they account for a significant proportion of the total costs of nursing home care [24].

By now, there is a considerable amount of literature on queueing models for capacity decisions in hospitals. The vast majority addresses issues related to medium and long-term capacity decisions, with a strong emphasis on bed occupancy, see e.g. [4, 5, 9, 13] and [10, 19, 22] for an overview. The central issue typically is to determine the required capacity (either in number of beds or staff) to accomodate the randomly arriving demand. On the short-term, randomly occuring patients needs lead to waiting for care delivery. The literature on short-term performance for clinical care is much more restrictive.

Mostly related to our setting are two studies proposing models for nurse staffing levels in hospital wards. Yankovic and Green [39] use a two-dimensional Markov model to describe nursing workload due to admissions and discharges in addition to the fluctuations in needs of patients that arise while a patient occupies a bed. They determine nurse staffing levels by evaluating the system performance numerically. Furthermore, they demonstrate that admission or discharge blocking caused by nurse shortage can have a significant impact on system performances and show that prespecified nurse-to-patient ratio policies cannot achieve a consistently high service level. De Véricourt and Jennings [8] study a similar model, but consider a fixed number of patients. Their analysis is based on many-server asymptotics. Their results also suggest that nurse-to-patient ratio policies cannot achieve a consistently high service level. Our study differs by the fact that we validate the model for short-term performance in a nursing home setting. Also, the model assumptions in [8, 39] differ slightly from ours. The performance analysis in [39] requires a more involved numerical procedure, whereas our formulas can be easily implemented in a decision support tool. The study of [8] considers an interesting asymptotic regime, which is however less relevant for our practical setting. Although queueing theory has been shown useful in assessing staffing levels in hospitals, the use of queueing models in guiding staffing decisions in a nursing home setting is still very limited.

2.2 Empirical context

From a queueing theoretic perspective, healthcare processes can be viewed as a system in which clients have to wait for the care they need, receive the necessary care and then depart [10]. The 'care on demand' process in a nursing home can be described as follows: when a nursing home resident needs care or support he/she pushes the call button in his/her room and waits until a care worker is available. Next, the available care worker moves to the room of the concerning nursing home resident. When the required care or assistance has been delivered, the resident leaves the 'care on demand' process (see Fig. 1). Queueing theory is an appropriate and useful method for modeling and analyzing this 'care on demand' process because it can handle the random character of call button requests and the variability in duration of healthcare tasks.

The Belgian nursing home under study provides long-term residential care for up to 180 clients who are aged 65 and over. Although all residents need some assistance with activities of daily living most of them are still largely self-sufficient. There are six care-providing departments, each of which are responsible for the care and support of a fixed number of residents. This nursing home uses a high-tech registration system for 'care on demand'. In particular, every call button request is registered automatically in a central data base. In addition, all care workers are equiped with a keycard. Every time a care worker enters or leaves the room of a resident, the keycard is swiped along an electronic keypad, registering the timestamp and the location.

In this study we focus on the 'care on demand' activities during the night shift. The care is provided by only a small number of care workers, as the total need for care or support is limited during this period of the day. The assigned care workers only have to handle call button requests which are being received in a single call center.

2.3 Performance measure

In order to make it possible for nursing home managers to monitor the performance of the 'care on demand' process they need a performance measure and an objective. A performance measure, as defined in this paper, refers to a metric used to quantify the efficiency and/or effectiveness of a process [27]. For 'care on demand' we find that from the standpoint of the client, waiting for care and/or support should be avoided as much as possible. Here, waiting refers to the time between call request and the moment a care worker is present, to be called *response time*. The response time should be below some threshold for most of the clients.

In line with other service sectors, such as call centers, we propose to measure response times in terms of service levels. Specifically, we define a targeted time window Y during which a care worker should be present at the client. The service level is defined as follows.

Definition The *service level* X/Y denotes that X % of the clients has a response time at or below Y minutes.

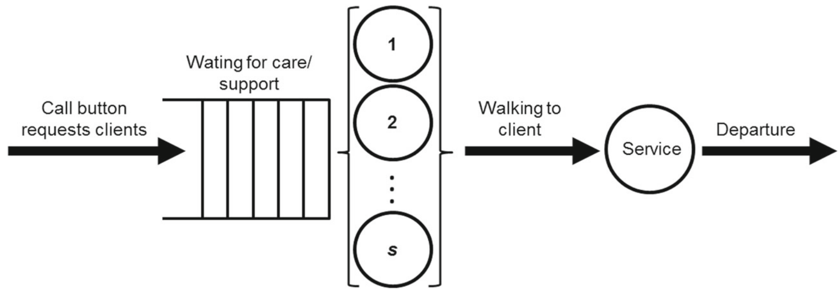


Fig. 1 The 'care on demand' queueing system

Based on Pareto's principle, a typical value for X is 80. Using a time window Y of 10 minutes then yields an 80/10 service level, meaning that in 80 % of the requests a care worker is present at the client within 10 minutes after the request is generated. The queueing model presented in this paper can be used as a tool to measure the performance of the 'care on demand' process and to determine the number of care workers required to meet a specific service level. Although performance management is widely used in the field of healthcare as a means to improve the quality and efficiency, this type of performance management for 'care on demand' processes in a nursing home context has received hardly any attention.

2.4 Outline of results

In this subsection we outline the results obtained in Sections 3–5. First, from the data analysis we conclude that the average demand patterns, i.e. call requests and care delivery durations, are stable during the night. During the day, there are some distinct peaks that are caused by 'care by appointment' activities. Different days of the week show similar demand patterns. For call requests we conclude that the interarrival times may be well approximated by an exponential distribution. For care delivery durations, the conclusions are less affirmative.

Based on the data analysis, we propose an M/G/s queueing model to determine service levels, where we also include travel times. This results in Eq. 5 that expresses the tail probability of the response time. We note that such an equation may be readily implemented in a decision support tool. Moreover, we validated the queueing model with the actual waiting time data. Some numerical experiments show that an 80/10 service level would work well for this nursing home facility.

3 Data analysis

To provide insight in the 'care on demand' activities, we analyze the arrival process of call button requests and the actual care delivery process.

3.1 The arrival process of call requests

In this study we analyze call button requests that are made during a period of three months, ranging from February 1, 2013 till May 1, 2013. During this period in total 19,996 requests arrived. As a first indication Fig. 2 shows the number of arrivals per day. Observe that the number of arrivals fluctuates over time: during March more call requests were made than during February and April. Unfortunately, the time window of the data set is restricted to three months excluding the option to be conclusive about seasonal pat-

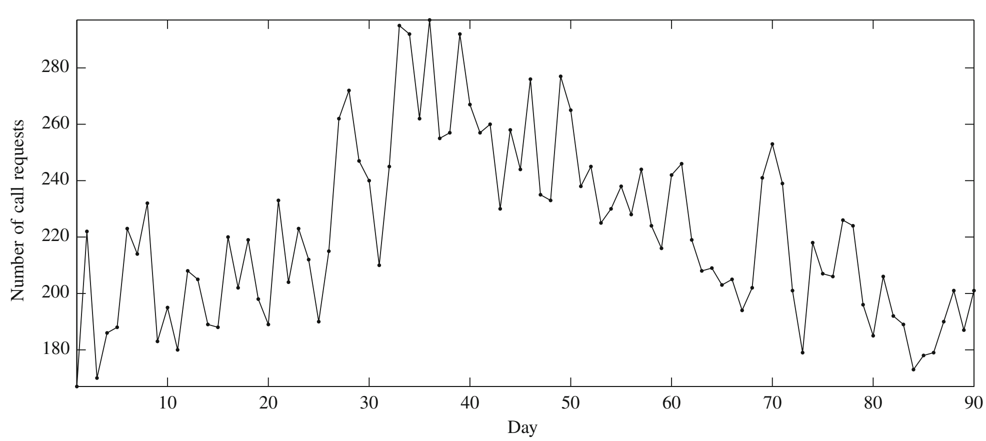


Fig. 2 Number of call requests per day

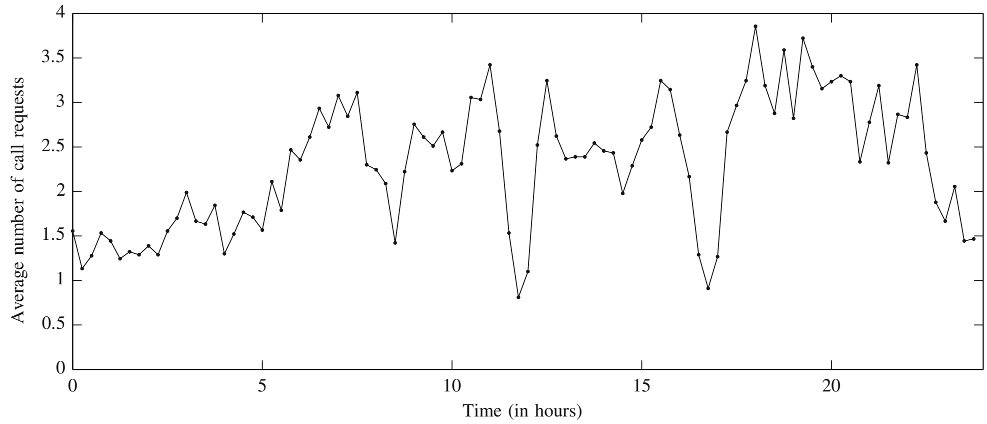


Fig. 3 Average number of call requests per quarter

terns. Given the state and mobility of residents, it seems likely that these fluctuations occur naturally; this idea is further supported by the huge variability in requests between residents and for requests over time.

Figure 3 shows the average number of arrivals per quarter. For instance, the first data point means that on average 1.6 call requests were made between 00:00 and 00:15. From this figure it can be seen that around 8:30, 12:00, and 17:00 on average less were made than during surrounding periods. These are moments in which the residents enjoy a joint activity like breakfast, lunch or diner, whereby they generate fewer calls. Moreover, it can be seen that the number of arrivals during the night[2] are fairly constant. In Fig. 4 a boxplot is given for the number of arrivals during each 15-minute interval. A data point that exceeds 1.5 times the interquartile range is defined as outlier, and is drawn as a circle. These boxplots confirm that the arrival rate over the course of a full day is inhomogeneous.

For each day of the week a similar arrival pattern has been observed, as shown in Fig. 5. Each line represents another weekday; it can be seen that the arrival patterns correspond with the overall arrival pattern, as shown in Fig. 3. This suggests that there is no structural weekly-pattern.

A common way to deal with the daily cycle of call requests is to consider intervals for which the number of arrivals is relatively stable. A prominent example of such a method is the stationary independent period-by-period (SIPP) approach, where the arrival rate is averaged over the staffing interval, see e.g. [14, 15]. We follow this method, but restrict the analysis to staffing decisions overnight. The two main reasons are that (i) the data are not compromised by 'care by appointment' related data, and (ii) in the day time, staff may not be solely dedicated to 'care on demand'.

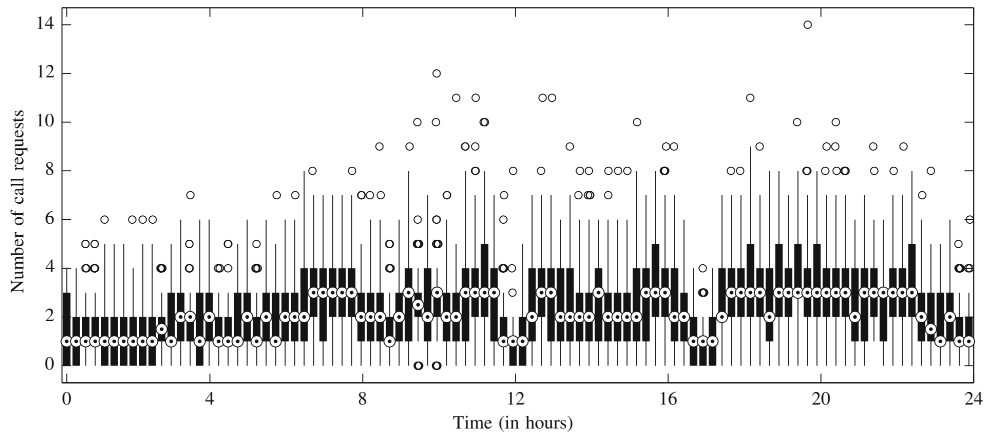
[2]The term 'night' refers to the time period between 22:45 and 5:45. This is the largest possible range in which the average number of arrivals per quarter does not exceed 2.2.

To confirm statistically whether the arrivals are constant during the night, i.e., between 22:45 and 5:45, the Kolmogorov-Smirnov test is used. This is done by testing for each combination of two 15-minute intervals the null hypothesis that the number of arrivals originate from the same underlying distribution. By using a significance level of 0.05 and applying the Bonferroni correction for the many tests that are done, this results in 0 out of 378 null hypotheses to be rejected. However, the Bonferroni correction is known to be conservative. A more powerful testing procedure is the positive false discovery rate (pFDR), introduced by Storey [32, 33]. This procedure also results in 0 null hypotheses to be rejected. Hence, it can be assumed that the arrival rate is constant during the night. In the remaining part of this paper we consider call requests that take place during this period, i.e. between 22:45 and 5:45. Other time intervals of the day can be analyzed in a similar way.

During the night, in total 3,891 call requests were made with an average interarrival time of 9.37 minutes. The interarrival time is here defined as the difference between the arrival times of two consecutive call requests. The standard deviation of the interarrival times is given by 9.83 and the coefficient of variation equals 1.05. In Fig. 6 (left) a histogram of the interarrival times is shown. From this figure it can be seen that the underlying distribution of the interarrival times corresponds with a right-tailed distribution. The exponential distribution, Gamma distribution and hyperexponential distribution have similar properties and are fitted to the data. The parameters of the exponential distribution and Gamma distribution are obtained by minimizing the mean squared error between the empirical and theoretical distribution, and the parameters of the hyperexponential distribution are obtained by using a three-moment fit, as given by Tijms [34].

For each fitted distribution, the Kolmogorov-Smirnov (KS) test is used to test the null hypothesis that the underlying distribution of the interarrival times is equal to the

Fig. 4 Boxplots of the number of call requests per quarter



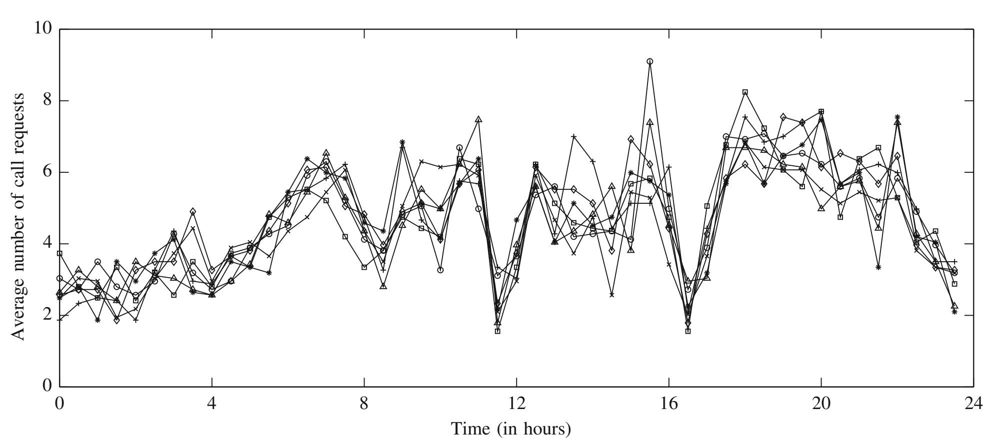
specified distribution. The estimated parameters and the p-values of the KS tests are given in Table 1. These results show that all of the fitted distributions are rejected, using a significance level of 0.05. This is not surprising given the considerable number of data points. To consider smaller sample sizes, we also conducted the KS test for the 15-minute intervals between 22:45 and 5:45 separately, resulting in 28 tests. On average, a 15-minute interval has 139 call button requests during these 3 months, which is a more suitable sample size for statistical testing. Using the KS test again and using a significance level of 0.05, the null hypothesis that the underlying distribution of the interarrival times is equal to the specified distribution is rejected for 11 intervals for the exponential and hyperexponential distribution and for all 28 intervals for the Gamma distribution. Based on the above, we may conclude that the interarrival times closely resemble an exponential distribution, but the match is not perfect. This is also confirmed by the exponential QQ-plot (which we omitted here).

In Fig. 6 a scaled probability density function (left) and cumulative distribution function (right) are shown of the interarrival times fitted with the exponential distribution. The plots of the Gamma distribution and hyperexponential distribution are similar. Figure 6 visually shows a good fit with the exponential distribution. Despite the fact that each of the fitted distributions are rejected by some of the KS tests, it seems practically useful to assume that the interarrival times are exponentially distributed.

3.2 Duration of care delivery

Upon a call button request, delivery of care is assumed to take place between the arrival and departure time of a care worker at the room of the client that did a request. In Fig. 7 the average time for care delivery per quarter is plotted. Clearly, between 6:45 and 9:45 these times are larger than during other periods of the day. Around this time the residents wake up and mainly receive 'care by appointment'. In practice, call requests around this time are often related to the scheduled activities, which causes higher care delivery times. For each 15-minute interval the average care delivery times are determined. Figure 8 shows for each 15-minute interval a boxplot of the average care delivery times, each based on roughly 90 data points. These boxplots show that the durations of care delivery are highly variable during certain periods of the day. Moreover, the average duration of care delivery is fairly stable across the night.

Fig. 5 Average number of call requests per weekday per quarter. Each line represents a different weekday



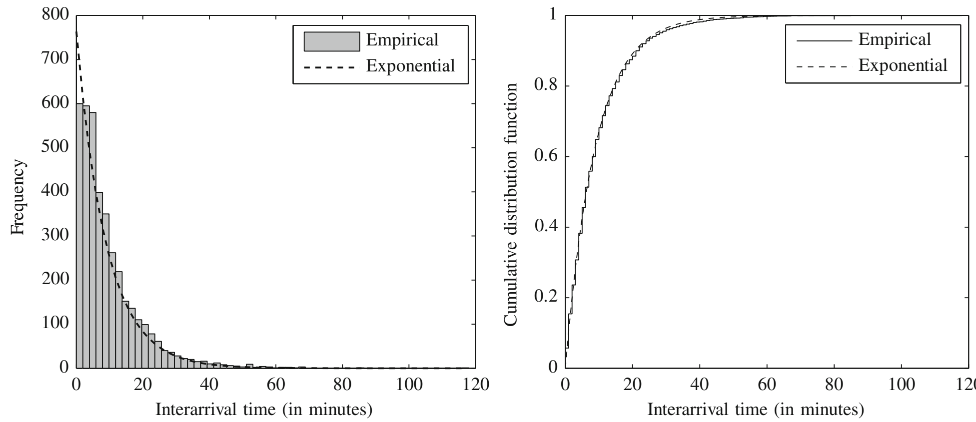


Fig. 6 Scaled probability density function (left) and cumulative distribution function (right) of the interarrival times fitted with the exponential distribution.

To estimate the distribution of the care delivery times, the same approach is used as in Section 3.1. The KS test combined with the Bonferroni correction confirms that in each combination of the 15-minute intervals between 22:45 and 5:45 the care delivery times originate from the same underlying distribution. When using a significance level of 0.05, 0 out of 378 null hypotheses are rejected. The same results are obtained when using positive false discovery rates.

During the night, in total 3,863 care delivery times were registered, with an average care delivery time of 2.56 minutes. The standard deviation of the care delivery times is 4.12 and the coefficient of variation is equal to 1.61. The histogram in Fig. 9 shows that the underlying distribution of the care delivery times is right-tailed. Obviously, a large number of call requests have a care delivery time below 1 minute. Probably, these are either 'false' requests that do not require assistance or are short questions. The coefficient of variation of 1.61 indicates that the underlying distribution of the care delivery times shows considerable variation, more than e.g. the exponential distribution. Nonetheless, the exponential distribution, Gamma distribution and hyperexponential distribution are fitted to the data; the estimated parameters and the p-values of the KS tests are given in Table 2. These parameters are estimated in the same way as in Section 3.1. The null hypothesis is tested that the underlying distribution of the care delivery times is equal to the specified distribution. At a significance level of 0.05, the null hypothesis is rejected. As in Section 3.1, we carried out the KS test for the 15-minute intervals separately resulting in 28 rejections out of 28 tests for all three distributions.

Based on the tests above, it is clear that the care delivery times do not match well to any of the proposed distributions from a statistical viewpoint. The hyperexponential distribution has the highest p-value, which is an indication that this distribution gives the best fit with the data compared to the other fitted distributions. This is also confirmed by plots of the scaled probability density function, cumulative distribution function and QQ-plot made for each of the distributions. Plots of the former two are given for the hyperexponential distribution in Fig. 9. These plots indicate that the hyperexponential distribution might be of some practical value. In that case, the parameters of the hyperexponential distribution as given in Table 2 can be interpreted as follows: with probability $\hat{p}_2 = 0.90$, the client has a minor request taken on average $1/\hat{\mu}_2 = 1.79$ minutes, whereas with probability $\hat{p}_1 = 0.10$ the client has a large request taking $1/\hat{\mu}_1 = 9.28$ minutes on average.

Table 1 Parameters and p-values for different distributions fitted with the interarrival times. The unit of time is minutes

Distribution	Parameters	p-value
Exponential	$\hat{\lambda} = 0.11$	1.47e-11
Gamma	$\hat{k} = 0.91$, $\hat{\theta} = 9.92$	3.02e-13
Hyperexponential	$\hat{p}_1 = 0.14$, $\hat{p}_2 = 0.86$, $\hat{\lambda}_1 = 0.07$, $\hat{\lambda}_2 = 0.12$	1.47e-11

4 Queueing model

The number of call requests and care delivery times are key ingredients for a model that may be applied to determine staffing levels overnight. Such a model should be useful for management at a strategic or tactical level. We approximate the service level of the proposed model in Section 4.1 and validate the model in Section 4.2.

4.1 Model and performance analysis

We identify three important features that the model should obey:

(i) The random nature of call requests and service times should be reflected in the model.
(ii) Time for traveling of care workers to a client should be taken into account.

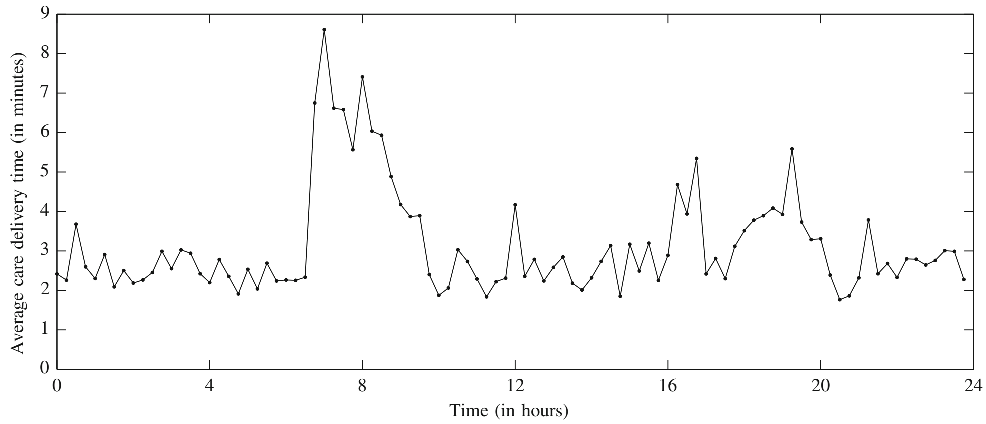


Fig. 7 Average care delivery time per quarter

(iii) The model should be sufficiently simple to be useful in practice.

Queueing models are the natural candidate in view of feature (i). We note that detailed simulation models may also capture the stochastic nature, but due to the limited availability of data and process information we opt for simple models that reflect the key characteristics of the health delivery process. Such simple models demonstrate the important principles for supporting staffing decisions on a strategic or tactical level and are sufficiently simple to implement.

Below, we discuss the elements of the queueing model.

Arrivals As discussed in Section 3.1, the interarrival times overnight are well approximated by an exponential distribution. The arrival of call requests are therefore assumed to follow a Poisson process with rate λ. We note that the number of residents is bounded, suggesting that a finite-source queueing model may be applicable when we assume that a resident does not generate any new calls when he/she is waiting. However, closed queueing models are much more difficult to analyze. Moreover, the number of residents is large enough (180 residents), such that the difference between open and closed models is negligible. For small-scale living facilities having in the order of 10 residents, some further analysis is required though, see also Remark 3.

Service times The time for care delivery is yet not entirely clear from Section 3.2. Moreover, the time required for care workers to react to call requests and travel to the corresponding room may be considerable. We refer to this combination as *travel time*. The service time S is then defined as $S = S_1 + S_2$, with S_1 the overall travel time and S_2 the time for care delivery. Unfortunately, information about travel times can not be derived from the data; such durations also highly depend on the local situation. In the light of (iii), we propose to use a two-phase hypoexponential distribution with parameters $\gamma_1 \neq \gamma_2$ for S_1, that is the sum of two exponential durations. The first phase may be interpreted as time to react (e.g. notice the call, finish current task) and the second phase as actual traveling time. The two-phase hypoexponential distribution has coefficient of variation between 0.5 and 1, and the peak in probability mass is at a point larger than zero if $\gamma_1, \gamma_2 < \infty$. We consider this approximation to be reasonable. Note that the above implies that the full service time S is general, as we did not yet make an assumption for S_2.

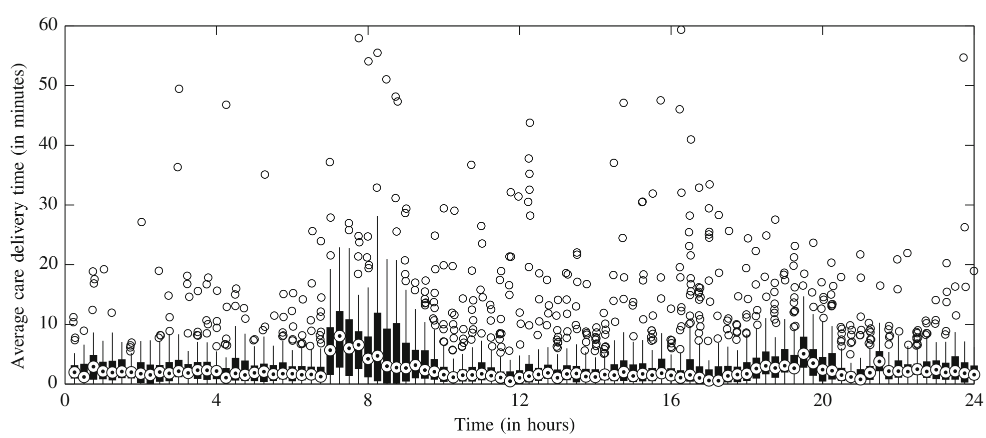


Fig. 8 Boxplots of the average care delivery time per quarter

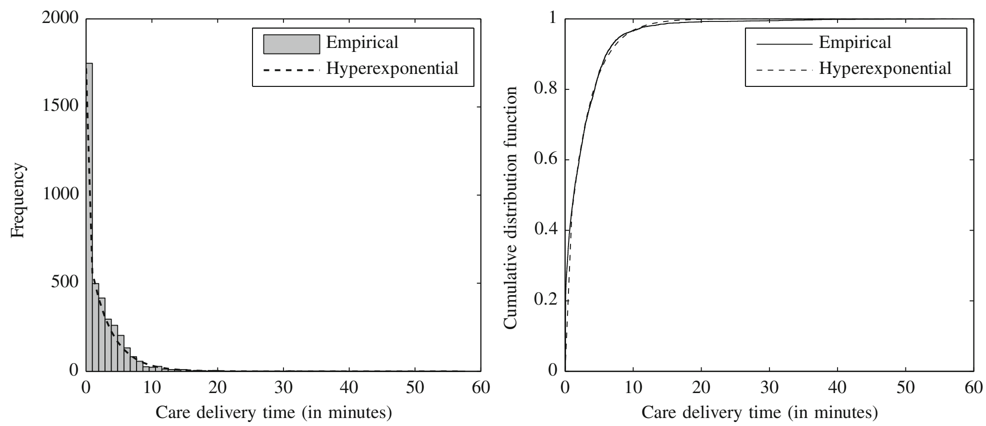


Fig. 9 Scaled probability density function (left) and cumulative distribution function (right) of the care delivery times fitted with the hyperexponential distribution

Servers Let s be the number of servers, representing the care workers. We assume that the care workers are dedicated to 'care on demand' tasks. This is reasonable overnight due to the limited 'care by appointment' activities. By day, it depends on how the care process is organized whether 'care on demand' and 'care by appointment' are mixed or separated.

The arguments above suggest to use the M/G/s queueing model. At this point we like to make two relevant remarks. First, the M/G/s model is intractable and for its analysis we rely on approximations available in the literature. Second, the waiting time in the M/G/s queue corresponds to the time when a care worker is available to visit a client. In line with the queueing literature, we refer to this as W_Q. The time that a client is actually waiting for a care worker includes traveling time, i.e., the performance measure of interest is $R = W_Q + S_1$, with R referring to *response time*.

We now first consider approximations for the waiting time W_Q in the M/G/s queue. An important point of reference is the M/M/s queue. As the coefficient of variation of S_2 is 1.61, see Section 3.2, and the coefficient of variation of S_1 is between 0.5 and 1, approximating the service time $S = S_1 + S_2$ by an exponential random variable may not be that bad (the coefficient of variation of S is likely not too far off from 1).

We follow the approximation proposed by Whitt [37, 38] and consider the probability of delay $\mathbb{P}(W_Q > 0)$ and the waiting time distribution separately. The probability of waiting in the M/G/s queue has been relatively well studied in the literature, see e.g. [20, 37, 38] and references therein. As noted in [38, p. 134] and [20, p. 371], the probability of waiting in the M/M/s model is usually an excellent approximation for the probability of waiting in the corresponding M/G/s queue. Hence we have

$$\mathbb{P}(W_Q > 0) \approx \frac{a^s}{(s-1)!(s-a)} \left[\sum_{i=0}^{s-1} \frac{a^i}{i!} + \frac{a^s}{(s-1)!(s-a)} \right]^{-1}, \quad (1)$$

where $a = \lambda \mathbb{E}S$ is the offered load. Let $\rho = a/s$ denote the load per care worker and assume that $\rho < 1$. In heavy traffic, the conditional waiting time $(W_Q | W_Q > 0)$ has an exponential distribution. Moreover, for the M/M/s queue the waiting time is also exponential. In line with [1, 20, 37] we suggest a simple exponential approximation of the waiting time given that the waiting time is positive. It remains to specify the paramter of this exponential distribution. We follow Whitt [37] and let the parameter coincide with heavy-traffic analysis, yielding

$$\mathbb{P}(W_Q > t) \approx \mathbb{P}(W_Q > 0) e^{-\beta t}, \quad (2)$$

with

$$\beta = \frac{2}{c_A^2 + c_S^2}(1-\rho)s. \quad (3)$$

Here the squared coefficient of variation of the interarrival times c_A^2 equals 1, because arrivals are assumed to follow a Poisson process; the squared coefficient of variation of the overall service time S is given by

$$c_S^2 = \frac{\mathbb{V}\mathrm{ar}S}{(\mathbb{E}S)^2} = \frac{\mathbb{V}\mathrm{ar}S_1 + \mathbb{V}\mathrm{ar}S_2}{(\mathbb{E}S_1 + \mathbb{E}S_2)^2} = \frac{1/\gamma_1^2 + 1/\gamma_2^2 + 16.94}{(1/\gamma_1 + 1/\gamma_2 + 2.56)^2},$$

assuming that S_1 and S_2 are independent and $\mathbb{E}S_2$ and $\mathbb{V}S_2$ follow from Section 3.2.

Now, we turn to the response time, which is the convolution of the waiting time with the travel time. Let $F_{S_1}(\cdot)$ be

Table 2 Parameters and p-values for different distributions fitted to the care delivery times. The unit of time is minutes

Distribution	Parameters	p-value
Exponential	$\hat{\mu} = 0.43$	2.24e-125
Gamma	$\hat{k} = 0.53, \hat{\theta} = 4.78$	1.56e-88
Hyperexponential	$\hat{p}_1 = 0.10, \hat{p}_2 = 0.90, \hat{\mu}_1 = 0.11,$ $\hat{\mu}_2 = 0.56$	1.59e-45

the distribution function of S_1, which is given by, for $t \geq 0$,

$$F_{S_1}(t) = 1 - \frac{1}{\gamma_2 - \gamma_1}\left(\gamma_2 e^{-\gamma_1 t} - \gamma_1 e^{-\gamma_2 t}\right). \tag{4}$$

Conditioning on the waiting time, and combining the results above provides an approximation for the quantity of interest: for $t \geq 0$,

$$\begin{aligned}
\mathbb{P}(R \leq t) &= \mathbb{P}(W_Q = 0) F_{S_1}(t) + \int_0^t F_{S_1}(t-u)\mathrm{d}\mathbb{P}(W_Q \leq u) \\
&\approx (1 - \mathbb{P}(W_Q > 0))\left[1 - \frac{1}{\gamma_2 - \gamma_1}\left(\gamma_2 e^{-\gamma_1 t}\right.\right. \\
&\quad \left.\left. - \gamma_1 e^{-\gamma_2 t}\right)\right] + \mathbb{P}(W_Q > 0) \\
&\quad \times \int_0^t \left(\beta e^{-\beta u} - \frac{\beta\gamma_2}{\gamma_2 - \gamma_1} e^{-\gamma_1 t} e^{-u(\beta - \gamma_1)}\right. \\
&\quad \left. + \frac{\beta\gamma_1}{\gamma_2 - \gamma_1} e^{-\gamma_2 t} e^{-u(\beta - \gamma_2)}\right) \mathrm{d}u \\
&= 1 - (1 - \mathbb{P}(W_Q > 0)) \frac{1}{\gamma_2 - \gamma_1}\left(\gamma_2 e^{-\gamma_1 t} - \gamma_1 e^{-\gamma_2 t}\right) \\
&\quad + \mathbb{P}(W_Q > 0)\left(\frac{-\gamma_1\gamma_2}{(\beta - \gamma_1)(\beta - \gamma_2)} e^{-\beta t}\right. \\
&\quad - \frac{\beta\gamma_2}{(\gamma_2 - \gamma_1)(\beta - \gamma_1)} e^{-\gamma_1 t} \\
&\quad \left. + \frac{\beta\gamma_1}{(\gamma_2 - \gamma_1)(\beta - \gamma_2)} e^{-\gamma_2 t}\right),
\end{aligned} \tag{5}$$

where we used Eqs. 4 and 2 in the second step and the third step follows from basic calculus. Here $\mathbb{P}(W_Q > 0)$ and β are given by Eqs. 1 and 3, respectively. This simple formula thus provides the approximate service level, i.e., the fraction of clients that wait no longer than t minutes for receiving care.

Remark 1 The most involved part in the performance analysis may be $\mathbb{P}(W_Q > 0)$. A more elementary approximation for this is due to Sakasegawa [29]

$$\mathbb{P}(W_Q > 0) \approx \rho^{\sqrt{2(s+1)}-1}.$$

As this approximation is mainly accurate for high loads, we advocate to use Eq. 1.

Remark 2 Kimura also suggests an exponential distribution for the conditional waiting time, but proposes a refined parameter β that is also exact in light traffic, see Equation (5.12) of [20]. Since the more elementary β above suffices, see also the next subsection, we advocate to use the simpler one here.

Remark 3 Assuming that the number of residents is fixed at N and that they do not generate any new calls during waiting, the relevant model is in fact the finite-source M/G/s//N system. For exponential service times the number of customers in such a system is a birth-and-death process from which the stationary distribution of the number of waiting customers π_i is easily derived, see for instance Kleinrock [21]. The waiting time distribution is then

$$\mathbb{P}(W_Q > t) = e^{-s\mu t} \sum_{k=s}^{n} \frac{(n-k)\pi_k}{\sum_{i=0}^{n}(n-i)\pi_i} \sum_{j=0}^{k-s} \frac{(s\mu t)^j}{j!}.$$

This also leads to closed-form results, but the convolution with the travel time is now more involved. For small-scale living facilities, the M/M/s//N model is more appropriate. Caution is required when the coefficient of variation is far off from 1. In that case, a viable option is to rely on relations between open and closed queueing systems, as in e.g. [30, 36].

4.2 Model validation

We validated the model using the time frame between 22:45 and 5:45 again. Most parameters can be derived from Section 3. For this nursing home we assumed the number of care workers fixed at three during the night. In practice, the number of care workers is seldomly fixed throughout the year due to illnesses, deficient scheduling during leaves, and other care activities such as ‘care by appointment’. As mentioned, the parameters γ_1 and γ_2 can not be estimated directly as there is no data available for travel times. We estimated the parameters γ_1 and γ_2 by minimizing the mean squared error between the empirical cumulative distribution function of the response times and $\mathbb{P}(R \leq t)$ as presented in Eq. 5. For $s = 3$, this yields $\hat{\gamma}_1 = 2.87$ and $\hat{\gamma}_2 = 0.20$. From practical experience, these values seem to resemble the total *travel time* reasonably well.

The probability density function (pdf) and the cumulative distribution function (cdf) of the response time R are displayed in Fig. 10 along with the empirical versions. Because the response time is the convolution of W_Q with the travel time S_1, it has no probability mass in zero. In other words, every client has to wait at least until a care worker is present in the room. Overall, the theoretical model fits fairly well to the empirical distribution. For very small response times, there is a difference in pdf. In particular, the peak of the empirical distribution is shifted a little to the right compared to our approximate queueing model. This indicates that the two-phase phase hypoexponential assumption for S_1 may not be perfect. For the cdf, the largest difference occurs for response time about twenty minutes, although the differences are modest. For the empirical distribution also a small peak seems to emerge around this time window.

To verify the approximation, we also simulated the finite source M/G/s//N model with $N = 180$ residents. For the

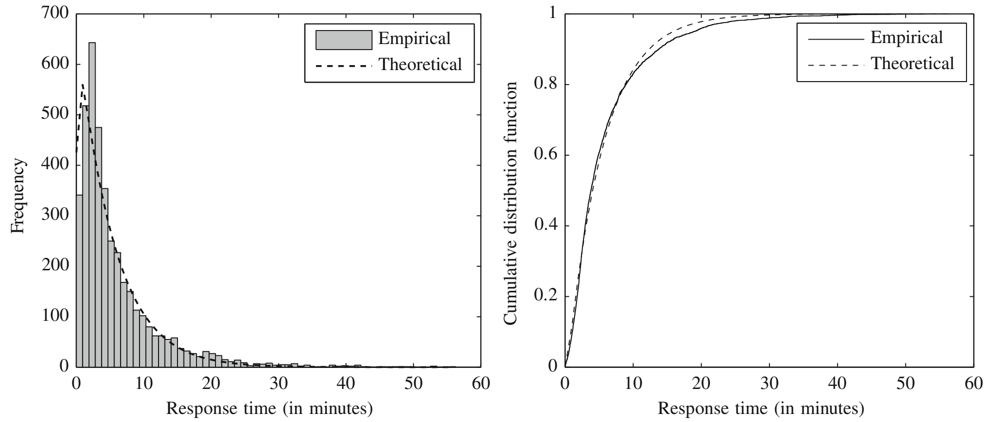


Fig. 10 Scaled probability density function (left) and cumulative distribution function (right) of the empirical response times with the response time distribution based on the M/G/3 model

travel time S_1, we used the two-phase hypoexponential distribution and for the care delivery time S_2, we randomly draw from the empirical data. The simulation results (5,000,000 care requests and a warm-up period of 100,000 care requests) are very similar to the response-time approximation. We compared the empirical distribution from the simulation with the approximate theoretical distribution function from Section 4.1. The mean relative error between these distributions is 0,1495 %. We omitted the simulation results in Fig. 10 as the lines for the simulation and the approximation are indistinguishable.

Our experience is that nursing homes are not managed based on service level agreements. However, a typical quantity of interest is the fraction of clients that wait no longer than ten minutes, which may be defined as the service level (see Section 2.3). For both the approximate queueing model and the data, this is slightly over 80 %. For such service levels, the approximation is rather accurate. Our general conclusion is that for any practical purposes the M/G/s model seems to suffice. More specifically, the service levels based on our M/G/s approximation are not far off from the realized service levels in the data. In the next section, we exploit the queueing model to evaluate the impact of different practical scenarios.

5 Numerical scenarios

In this section we use the approximate queueing model to obtain insight in different nursing home scenarios. Specifically, we investigate the impact of care delivery times, call requests and scale on the response time R. As introduced in Section 2.3, we focus on the service level (SL) as our performance measure. A SL of X/Y denotes that

$$\mathbb{P}(R \leq Y) = \frac{X}{100},$$

that is, the response time is below Y minutes for X percent of the clients. In the current situation an 80/10 SL is met as the probability that the response time is larger than 10 minutes is slightly below 0.2.

In view of the changing landscape for long-term care, nursing homes will increasingly face questions related to capacity decisions and retaining nursing staff. As noted in [3, 23], and references therein, elderly people living in long-term care facilities have increasingly complex care needs. This may manifest in longer durations of care delivery, an increased number of call button requests, or both. In turn, this will affect appropriate staffing levels. Below, we briefly investigate the relations between service levels and the intensity of care.

For all figures, we use the situation and parameters as described in Section 4 as our basic scenario. In Fig. 11, we plotted the response time Y for different service levels as the mean duration of care delivery $\mathbb{E}S_2$ varies (left) or the number of call requests λ varies (right). For example, for the current situation the mean care delivery time is 2.56 minutes and on average 1.6 call requests arrive per quarter. We can read from the vertical axis that a 60 % SL is achieved at roughly 5.2 minutes, i.e. $\mathbb{P}(R \leq 5.2) \simeq 0.6$. The 70, 80 and 90 % service levels are achieved at approximately 6.7, 8.8, and 12.4 minutes. As such, for every mean care delivery time (left) and arrivals of call requests (right), the impact of different choices for the SL target may be read along the vertical axis. For the SL displayed the difference between a SL of 80 and 90 % is the largest, which resembles that the target time is increasing faster as the fraction of clients that should meet the response target increases. In other words, the amount of *extra* capacity requirements are increasing as the SL becomes more tight.

Reading Fig. 11 along the horizontal axis, you may see the impact of increasing the mean time for care delivery (left) and intensity of call requests (right). The impact on the SL is rather modest as the parameters change slightly (which is due to the relatively small load of the system). Even if $\mathbb{E}S_2$ is 10 minutes, then in 80 % of the cases a care worker is present within 15 minutes. Note that the impact is largest for the 90 % SL.

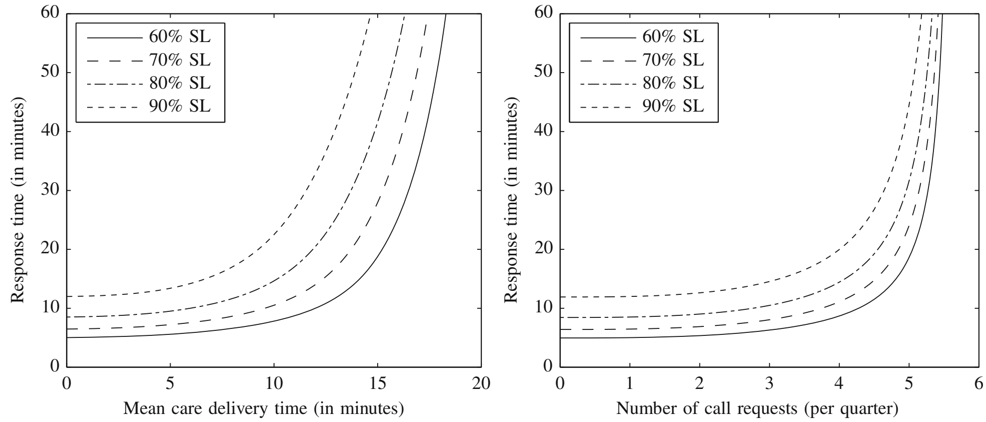


Fig. 11 The impact of mean care delivery time (left) and intensity of call requests per quarter (right) on the response time Y to achieve some fixed service level $X \in \{60, 70, 80, 90\}$

Figure 12 shows the relation between the SL (for four different target response times Y) and mean care delivery time. For instance, for the current mean care delivery time of 2.56 minutes and a target time of 5 minutes the SL is only 59 %, whereas the SL is 84, 94 and 98 % for target times Y of 10, 15, and 20 minutes, respectively. From the above it follows directly that a target time of 5 minutes is not very useful. Even if the mean care delivery time is negligible, the SL is then still only 60 %. This is due to the traveling time. Again, for a fixed mean care delivery time, the SL for different response time targets may be read along the vertical axis. Using these curves a target in the range of 10-15 minutes seems appropriate. Reading Fig. 12 along the horizontal axis, it can be seen which mean care delivery times can be handled while maintaining a specified SL target. As an example, the mean care delivery time may rise up to about 6 minutes to maintain an 80/10 SL. Based on the insights from the figures above, we advocate to use 80/10 SL for this particular situation.

Finally, we investigate the effect of scale. The mean care delivery time is fixed as in the basic scenario. As starting point, we assume that an 80/10 SL is desirable. We vary the scale in terms of number of care workers (employees) and let the arrival rate of call requests vary accordingly, such that the 80/10 SL is met exactly. A difficult issue is the impact of scale on traveling times. It seems natural that traveling times increase as scale increases. To what extent this holds strongly depends on the local condition as design of the building and relative positions of different units. Moreover, for longer distances other types of transport (e.g. scooters) may be profitable, such that drawbacks related to scale may be circumvented. Here, we consider four scenarios where the mean travel time $\mathbb{E}S_1$ increases with 0, 10, 20, and 50 % of the mean rate of call requests.

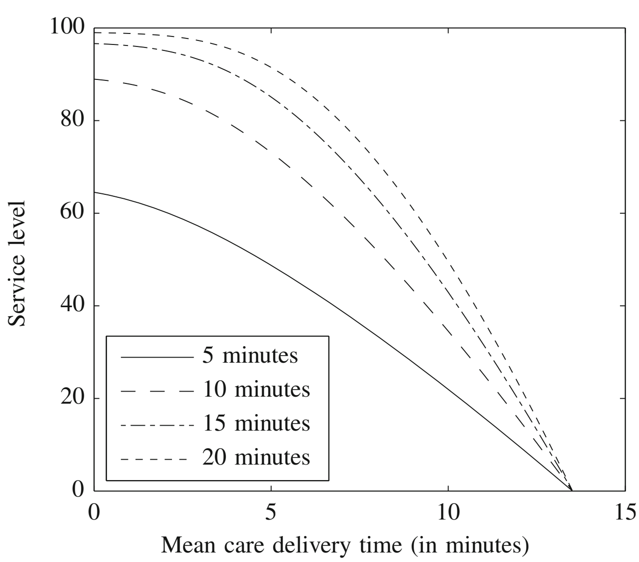


Fig. 12 The impact of mean care delivery time on the service level X for different values of the response target $Y \in \{5, 10, 15, 20\}$

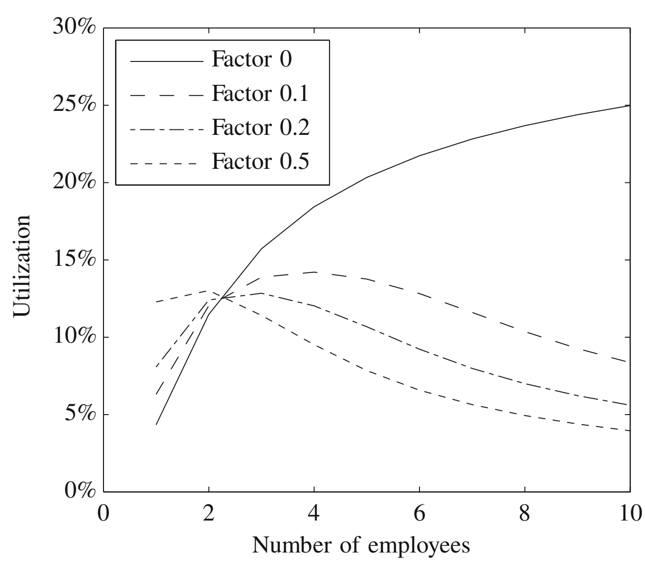


Fig. 13 Impact of scale (number of employees) on utilization excluding traveling to achieve an 80/10 SL

In Fig. 13 we illustrate the impact of scale on the utilization for the four traveling time . The utilization is based on times of care delivery and excludes travel times. The low utilization follows from considerable travel times, requiring overcapacity. When mean travel times remain constant (factor is 0), we see that the utilization increases in a concave manner. This is in line with the general concept of economies of scale. At larger scale, the relative variability in required care decreases and the utilization increases to maintain the same SL. This effect however becomes weaker when scale increases. Hence, 'care on demand' should not be organized at a too small scale.

In case travel times increase with scale (factor larger than 0), there is a trade-off between traditional economies of scale and impact of travel times. From Fig. 13, we see that the optimal size depends on the specific factor and, therefore, on the local situation. In any case, the optimal number of employees is at least two confirming that organizational units for 'care on demand' should not be too small.

6 Conclusions and discussion

In this paper we made a first step in trying to understand the real-life performance of the 'care on demand' process in a Belgian nursing home facility using a queueing theoretic approach. From a methodological point of view, the contribution of this study is twofold. First, by using real-life data we obtain insight in the number of call requests and care delivery times for 'care on demand' activities. Secondly, we developed a queueing model to support capacity decisions. Based on numerical experiments, we propose an 80/10 service level for this specific nursing home facility, meaning that at least 80 % of the clients should receive care within 10 minutes after a call button request. Although we think that an 80/10 service level will be suitable in many situations, this may depend on the specific context.

From a practical perspective, this study provides a basis on which it is possible to develop a staffing support tool for 'care on demand' activities which would allow nursing home managers to 1) determine the number of care workers required to sufficiently meet the needs and preferences of the nursing home residents when it comes to 'care on demand' and 2) to better understand the implications of their decisions (i.e. what-if scenarios). We think that such a tool has the potential to make an important contribution in the quest for more efficiency, without losing sight of the needs of residents.

A model is never a complete representation of reality and the queueing model presented in this paper is no exception. First of all, this study is limited in scope because it only addresses the night care. A similar approach could be taken for the 'care on demand' process during day time. During day time, the amount of 'care by appointment' activities are much larger compared to the night, which may lead to compromised 'care on demand' data. Moreover, data on traveling times is lacking. Although travel times may vary depending on the local situation, it would be of interest to model this in more detail. Finally, we used an approximation for the queueing model. This approximation is expected to work well in most nursing-home situations, but the accuracy may decrease when e.g. the number of clients is getting very small.

Despite the fact that long-term elderly care will become increasingly important in the next decades, the body of operations research (OR) literature directed on this topic is still very limited. Therefore we would like to challenge researchers in the field of OR to put more emphasis on research in long-term elderly care. Finding usable data will be an important first step for future research in this promising field, as reliable and valid information is scarce and seldomly collected. Nevertheless, the most important challenge for future research will be to not overemphasize the importance of efficiency as the needs and preferences of the clients should always be kept in mind when conducting research in this area.

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